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1 Basics of Geometry

WATCH INFO

NATIONAL GEOGRAPHIC EXPLORER

Rae Wynn-Grant

Dr. Rae Wynn-Grant is an ecologist who uses statistical modeling to investigate how anthropogenic factors can influence the spatial patterns of carnivore behavior and ecology. She studies the ecological and social drivers of human-carnivore conflict.

- What is a carnivore? Name several large carnivores that live in North America.
- Ecology is the branch of biology that deals with relationships among animals. Give several examples of predator-prey relationships in North America.

STEM

When a carnivore's habitat is diminished, the likelihood of human-carnivore conflict increases. In the Performance Task, you will design a wildlife reservation to provide a protected habitat for a tiger population.

Ecology

Preparing for Chapter 1

Chapter Learning Target Understand basics of geometry.

Chapter Success Criteria
- ◆ I can name points, lines, and planes.
- ◆ I can measure segments and angles.
- ■ I can use formulas in the coordinate plane.
- ■ I can construct segments and angles.

◆ Surface
■ Deep

Chapter Vocabulary

Work with a partner. Discuss each of the vocabulary terms.

point	acute angle
line	right angle
plane	obtuse angle
line segment	complementary angles
angle	supplementary angles

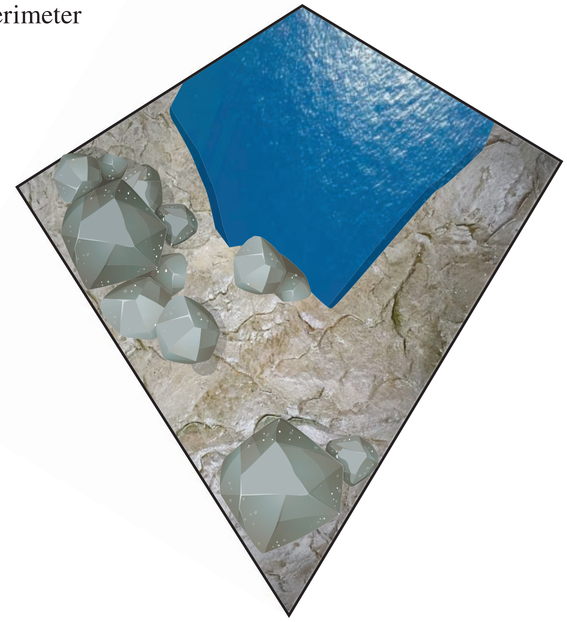
Mathematical Practices

Make Sense of Problems and Persevere in Solving Them

Mathematically proficient students plan a solution pathway rather than simply jumping into a solution attempt.

Work with a partner. The figure shown represents a polar bear enclosure at a zoo, where 1 centimeter represents 25 feet.

1. What information do you need in order to find the perimeter of the enclosure? Explain how you can find this information. Then find the perimeter.
2. What information do you need in order to find the area of the enclosure? Explain how you can find this information. Then find the area.



1 Prepare WITH CalcChat®

Finding Absolute Value

Example 1 **Simplify $|-7 - 1|$.**

$$|-7-1| = |-7+(-1)| \quad \text{Add the opposite of 1.}$$
$$= |-8| \quad \text{Add.}$$
$$= 8 \quad \text{Find the absolute value.}$$

▶ $|-7-1| = 8$

Simplify the expression.

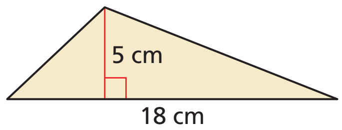
1. $|8-12|$ **2.** $|-6-5|$ **3.** $|4+(-9)|$

4. $|13+(-4)|$ **5.** $|6-(-2)|$ **6.** $|5-(-1)|$

7. $|-8-(-7)|$ **8.** $|8-13|$ **9.** $|-14-3|$

Finding the Area of a Triangle

Example 2 **Find the area of the triangle.**

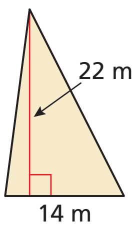


$$A = \frac{1}{2}bh \quad \text{Write the formula for area of a triangle.}$$
$$= \frac{1}{2}(18)(5) \quad \text{Substitute 18 for } b \text{ and 5 for } h.$$
$$= \frac{1}{2}(90) \quad \text{Multiply 18 and 5.}$$
$$= 45 \quad \text{Multiply } \tfrac{1}{2} \text{ and 90.}$$

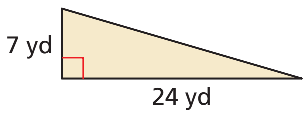
▶ The area of the triangle is 45 square centimeters.

Find the area of the triangle.

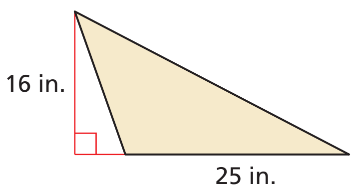
10.



11.



12.



13. **MP REASONING** Describe the possible values for x and y when $|x-y| > 0$. What does it mean when $|x-y| = 0$? Can $|x-y| < 0$? Explain your reasoning.

1.1 Points, Lines, and Planes

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Learning Target Use defined terms and undefined terms.

Success Criteria
- I can describe a point, a line, and a plane.
- I can define and name segments and rays.
- I can sketch intersections of lines and planes.

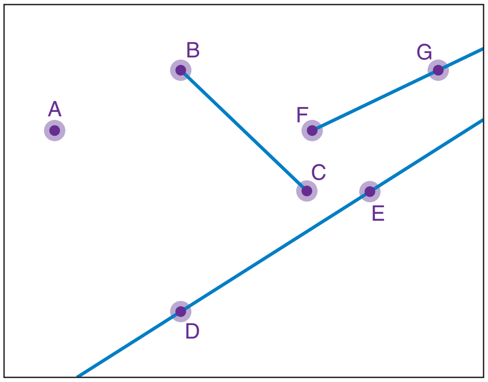
EXPLORE IT! Using Technology

Math Practice

Listen and Ask Questions
Ask a few classmates to read their answers to parts (b)–(d). Ask any questions you have about their answers.

Work with a partner.

a. Use technology to draw several points. Also, draw some lines, line segments, and rays.

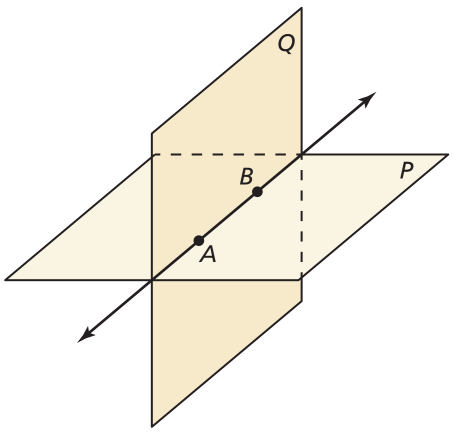


b. How would you describe a line? a point?

c. What is the difference between a line and a line segment? a line and a ray?

d. Write your own definitions for a line segment and a ray, based on how they relate to a line.

e. The diagram shows plane P and plane Q intersecting. How would you describe a plane?



f. MP **CHOOSE TOOLS** Describe the ways in which each of the following can intersect and not intersect. Provide a sketch or use real-life objects to model each type of intersection.

i. two lines

ii. a line and a plane

iii. two planes

Using Undefined Terms

In geometry, the words *point*, *line*, and *plane* are **undefined terms**. These words do not have formal definitions, but there is agreement about what they mean.

Vocabulary

undefined terms, *p. 4*
point, *p. 4*
line, *p. 4*
plane, *p. 4*
collinear points, *p. 4*
coplanar points, *p. 4*
defined terms, *p. 5*
line segment, or segment, *p. 5*
endpoints, *p. 5*
ray, *p. 5*
opposite rays, *p. 5*
intersection, *p. 6*

KEY IDEA

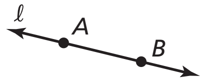
Undefined Terms: Point, Line, and Plane

Point A **point** has no dimension. A dot represents a point.

point *A*

Line A **line** has one dimension. It is represented by a line with two arrowheads, but it extends without end.

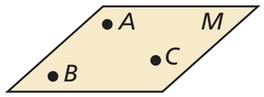
Through any two points, there is exactly one line. You can use any two points on a line to name it.



line ℓ, line *AB* ($\overleftrightarrow{AB}$), or line *BA* ($\overleftrightarrow{BA}$)

Plane A **plane** has two dimensions. It is represented by a shape that looks like a floor or a wall, but it extends without end.

Through any three points not on the same line, there is exactly one plane. You can use three points that are not all on the same line to name a plane.



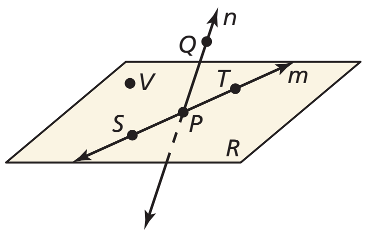
plane *M*, or plane *ABC*

Collinear points are points that lie on the same line. **Coplanar points** are points that lie in the same plane.

EXAMPLE 1 Naming Points, Lines, and Planes

a. Give two other names for $\overleftrightarrow{PQ}$ and plane *R*.

b. Name three points that are collinear. Name four points that are coplanar.



SOLUTION

a. Other names for $\overleftrightarrow{PQ}$ are $\overleftrightarrow{QP}$ and line *n*. Other names for plane *R* are plane *SVT* and plane *PTV*.

b. Points *S*, *P*, and *T* lie on the same line, so they are collinear. Points *S*, *P*, *T*, and *V* lie in the same plane, so they are coplanar.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1. Use the diagram in Example 1. Give two other names for $\overleftrightarrow{ST}$. Name a point that is *not* coplanar with points *Q*, *S*, and *T*.

2. **WRITING** Compare collinear points and coplanar points.

Using Defined Terms

In geometry, terms that can be described using known words such as *point* or *line* are called **defined terms**.

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KEY IDEA

Defined Terms: Segment and Ray

The diagrams below use the points A and B and parts of the line AB.

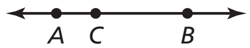
Segment A **line segment**, or **segment**, is a part of a line that consists of two **endpoints** and all points on the line between the endpoints.

segment AB ($\overline{AB}$), or segment BA ($\overline{BA}$)

Ray A **ray** is a part of a line that consists of an endpoint and all points on the line on one side of the endpoint.

ray AB ($\overrightarrow{AB}$)

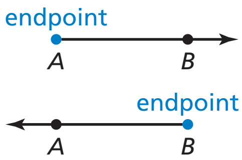
Opposite Rays Two rays that have the same endpoint and form a line are **opposite rays**.



$\overrightarrow{CA}$ and $\overrightarrow{CB}$ are opposite rays.

STUDY TIP

Note that $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are different rays.



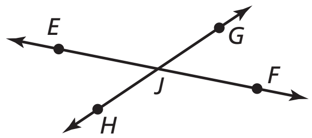
Segments and rays are collinear when they lie on the same line. So, opposite rays are collinear. Lines, segments, and rays are coplanar when they lie in the same plane.

EXAMPLE 2 Naming Segments, Rays, and Opposite Rays

WATCH

a. Give another name for $\overline{GH}$.

b. Name all rays with endpoint J. Which of these rays are opposite rays?



SOLUTION

a. Another name for $\overline{GH}$ is $\overline{HG}$.

b. The rays with endpoint J are $\overrightarrow{JE}$, $\overrightarrow{JG}$, $\overrightarrow{JF}$, and $\overrightarrow{JH}$. The pairs of opposite rays with endpoint J are $\overrightarrow{JE}$ and $\overrightarrow{JF}$, and $\overrightarrow{JG}$ and $\overrightarrow{JH}$.

COMMON ERROR

In Example 2, $\overrightarrow{JG}$ and $\overrightarrow{JF}$ have a common endpoint, but they are not collinear. So, they are *not* opposite rays.

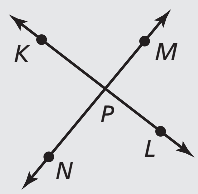
SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the diagram.

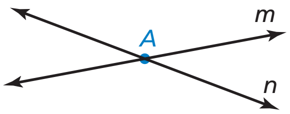
3. Give another name for $\overline{KL}$.

4. Are $\overrightarrow{KP}$ and $\overrightarrow{PK}$ the same ray? Are $\overrightarrow{NP}$ and $\overrightarrow{NM}$ the same ray? Explain.

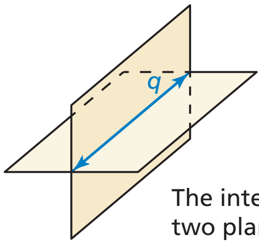


Sketching Intersections

Two or more geometric figures *intersect* when they have one or more points in common. The **intersection** of the figures is the set of points the figures have in common. Some examples of intersections are shown below.



The intersection of these two lines is a point.



The intersection of these two planes is a line.

EXAMPLE 3 Sketching Intersections of Lines and Planes

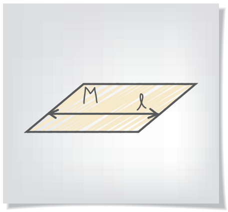
a. Sketch a plane and a line that is in the plane.

b. Sketch a plane and a line that does not intersect the plane.

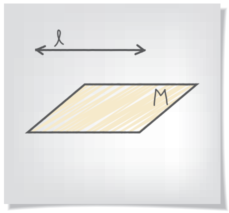
c. Sketch a plane and a line that intersects the plane at a point.

SOLUTION

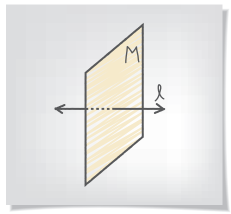
a.



b.

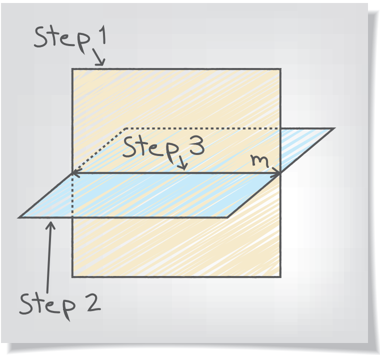


c.



EXAMPLE 4 Sketching an Intersection of Planes

Sketch two planes that intersect in a line.



SOLUTION

Step 1 Draw a vertical plane. Shade the plane.

Step 2 Draw a second plane that is horizontal. Shade this plane a different color. Use dashed lines to show where planes are hidden.

Step 3 Draw the line of intersection.

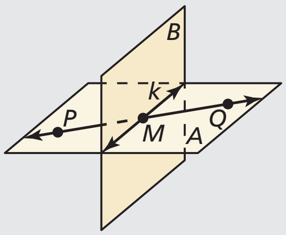
SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

5. Sketch two different lines that intersect a plane at the same point.

6. Sketch two planes that do not intersect.

Use the diagram.



7. Name the intersection of $\overleftrightarrow{PQ}$ and line k.

8. Name the intersection of plane A and plane B.

9. Name the intersection of line k and plane A.

Electric utilities use sulfur hexafluoride as an insulator. Leaks in electrical equipment contribute to the release of sulfur hexafluoride into the atmosphere.

Solving Real-Life Problems

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EXAMPLE 5 Modeling Real Life

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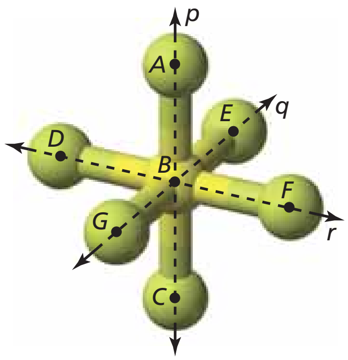
The diagram shows a model of a molecule of sulfur hexafluoride, the most potent greenhouse gas in the world. Name two different planes that contain line *r*.



SOLUTION

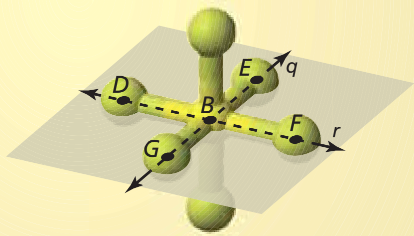
To name a plane that contains line *r*, use two points on line *r* and one point not on line *r*. Points *D* and *F* lie on line *r*. Points *C* and *E* do not lie on line *r*.

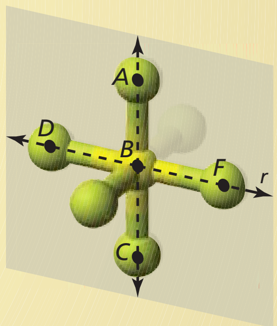
▶ So, plane *DEF* and plane *CDF* both contain line *r*.

COMMON ERROR

Because point *B* also lies on line *r*, you cannot use points *D*, *B*, and *F* to name a single plane. There are infinitely many planes that pass through these points.

Check The question asks for two *different* planes. Check whether plane *DEF* and plane *CDF* are two unique planes or the same plane named differently. Because point *C* does not lie in plane *DEF*, plane *DEF* and plane *CDF* are different planes.





SELF-ASSESSMENT

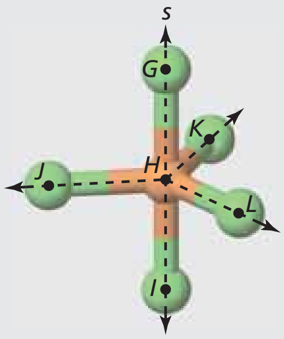
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use the diagram that shows a model of a molecule of phosphorus pentachloride.

10. Name two different planes that contain line *s*.

11. Name three different planes that contain point *K*.

12. Name two different planes that contain $\overrightarrow{HJ}$.



1.1 Practice WITH CalcChat® AND CalcView®

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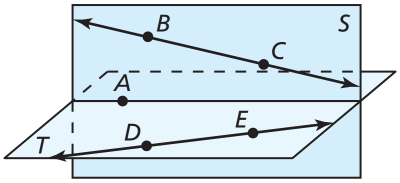
In Exercises 1–4, use the diagram.



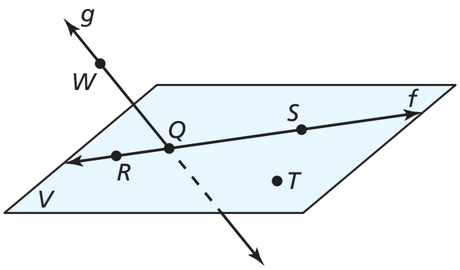
1. Name four points.

2. Name two lines.

3. Name the plane that contains points A, B, and C.

4. Name the plane that contains points A, D, and E.

In Exercises 5–8, use the diagram. *Example 1*



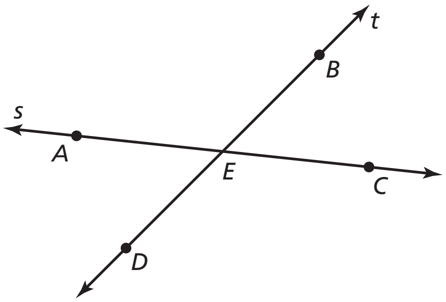
5. Give two other names for $\overleftrightarrow{WQ}$.

6. Give another name for plane V.

7. Name three points that are collinear. Then name a fourth point that is not collinear with these three points.

8. Name a point that is not coplanar with R, S, and T.

In Exercises 9–14, use the diagram. *Example 2*



9. What is another name for $\overline{BD}$?

10. What is another name for $\overline{AC}$?

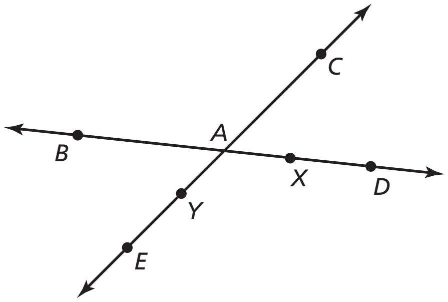
11. What is another name for $\overrightarrow{AE}$?

12. Name all rays with endpoint E.

13. Name two pairs of opposite rays.

14. Name one pair of rays that are not opposite rays.

ERROR ANALYSIS In Exercises 15 and 16, describe and correct the error in naming opposite rays in the diagram.



15. ✗ $\overrightarrow{AD}$ and $\overrightarrow{AC}$ are opposite rays.

16. ✗ $\overline{YC}$ and $\overline{YE}$ are opposite rays.

In Exercises 17–24, sketch the figure described.
Examples 3 and 4

17. plane P and line ℓ intersecting at one point

18. plane K and line m intersecting at all points on line m

19. $\overrightarrow{AB}$ and $\overleftrightarrow{AC}$

20. $\overrightarrow{MN}$ and $\overrightarrow{NX}$

21. plane M and $\overrightarrow{NB}$ intersecting at point B

22. plane M and $\overrightarrow{NB}$ intersecting at point A

23. plane A and plane B not intersecting

24. plane C and plane D intersecting at $\overleftrightarrow{XY}$

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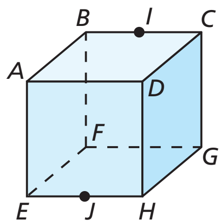
In Exercises 25–32, use the diagram.



25. Name a point that is collinear with points *E* and *H*.

26. Name a point that is collinear with points *B* and *I*.

27. Name a point that is not collinear with points *E* and *H*.

28. Name a point that is not collinear with points *B* and *I*.

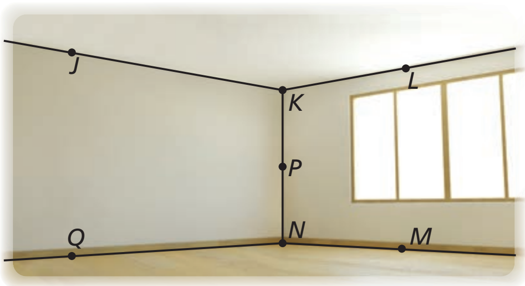
29. Name a point that is coplanar with points *D*, *A*, and *B*.

30. Name a point that is coplanar with points *C*, *G*, and *F*.

31. Name the intersection of plane *AEH* and plane *FBE*.

32. Name the intersection of plane *BGF* and plane *HDG*.

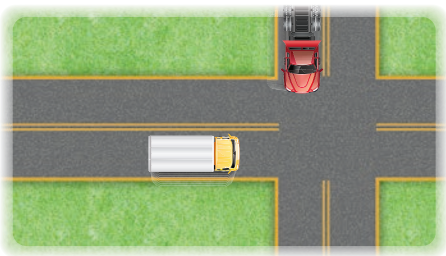
MODELING REAL LIFE **In Exercises 33 and 34, use the diagram.** *Example 5*



33. Name two points that are collinear with *P*.

34. Name two planes that contain *J*.

35. **MODELING REAL LIFE** When two trucks traveling in different directions approach an intersection at the same time, one of the trucks must change its speed or direction to avoid a collision. Two airplanes, however, can travel in different directions and cross paths without colliding. Explain how this is possible.



36. **CRITICAL THINKING** Given two points on a line and a third point not on the line, is it possible to draw a plane that includes the line and the third point? Explain your reasoning.

In Exercises 37–40, name the geometric term modeled by the part of the object indicated with an arrow.

37.

38.

39.

40.

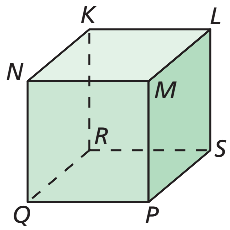
In Exercises 41–44, use the diagram to name all the points that are *not* coplanar with the given points.

41. *N*, *K*, and *L*

42. *P*, *Q*, and *N*

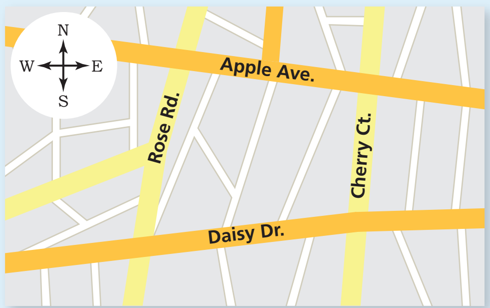
43. *P*, *Q*, and *R*

44. *R*, *K*, and *N*



45. **CRITICAL THINKING** Is it possible to draw two planes that intersect at one point? Explain your reasoning.

46. **HOW DO YOU SEE IT?**
You and your friend walk in opposite directions, forming opposite rays. You were originally on the corner of Apple Avenue and Cherry Court.



a. Name two possibilities of the road and direction you and your friend may have traveled.

b. Your friend claims he went north on Cherry Court, and you went east on Apple Avenue. Make an argument for why you know this could not have happened.

47. **MP REASONING** Explain why a four-legged chair may rock from side to side even if the floor is level. Would a three-legged chair on the same level floor rock from side to side? Why or why not?

48. **MODELING REAL LIFE** You are designing a living room. Counting the floor, walls, and ceiling, you want the design to contain at least eight different planes. Draw a diagram of your design. Label each plane in your design.

CONNECTING CONCEPTS In Exercises 49 and 50, graph the inequality on a number line. Tell whether the graph is a *segment*, a *ray* or *rays*, a *point*, or a *line*.

49. $x \leq 3$

50. $-7 \leq x \leq 4$

CRITICAL THINKING In Exercises 51–58, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.

51. A line ____________ has endpoints.

52. A line and a point ____________ intersect.

53. A plane and a point ____________ intersect.

54. Two planes ____________ intersect in a line.

55. Two points ____________ determine a line.

56. Any three points ____________ determine a plane.

57. Any three points not on the same line ____________ determine a plane.

58. Two lines that are not parallel ____________ intersect.

59. **MP STRUCTURE** Two coplanar intersecting lines will always intersect at one point. What is the greatest number of intersection points that exist if you draw four coplanar lines? Explain.

60. **THOUGHT PROVOKING**
Is it possible for three planes to never intersect? to intersect in one line? to intersect in one point? Sketch the possible situations.

GO DIGITAL

REVIEW & REFRESH

WATCH

In Exercises 61 and 62, determine which of the lines, if any, are parallel or perpendicular. Explain.

61. Line a passes through $(1, 3)$ and $(-2, -3)$.
Line b passes through $(-1, -5)$ and $(0, -3)$.
Line c passes through $(3, 2)$ and $(1, 0)$.

62. Line a: $y + 4 = \frac{1}{2}x$
Line b: $2y = -4x + 6$
Line c: $y = 2x - 1$

In Exercises 63 and 64, solve the equation.

63. $18 + x = 43$

64. $x - 23 = 19$

65. **MODELING REAL LIFE** You bike at a constant speed of 10 miles per hour. You plan to bike 30 miles, plus or minus 5 miles. Write and solve an equation to find the minimum and maximum numbers of hours you bike.

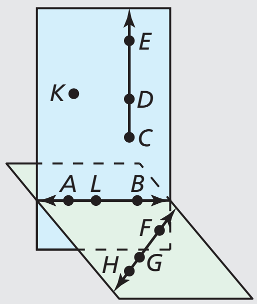
In Exercises 66 and 67, evaluate the expression.

66. $\sqrt[3]{8^5}$

67. $36^{1/2}$

68. Graph $f(x) = -\frac{1}{3}x + 5$ and $g(x) = f(x - 4)$. Describe the transformation from the graph of f to the graph of g.

In Exercises 69–75, use the diagram.



69. Name four points.

70. Name two lines.

71. Name three rays.

72. Name three collinear points.

73. Name three coplanar points.

74. Give two names for the plane shaded blue.

75. Name three line segments.

In Exercises 76–79, use zeros to graph the function.

76. $y = 2x(x - 5)(x + 8)$

77. $y = 4x^3 - 64x$

78. $y = 3x^3 + 3x^2 - 6x$

79. $y = -x(x + 1)(x - 7)$

In Exercises 80 and 81, make a box-and-whisker plot that represents the data.

80. Scores on a test: 76, 90, 84, 97, 82, 100, 92, 90, 88

81. Minutes spent at the gym: 60, 45, 50, 45, 65, 50, 55, 60, 60, 50

1.2 Measuring and Constructing Segments

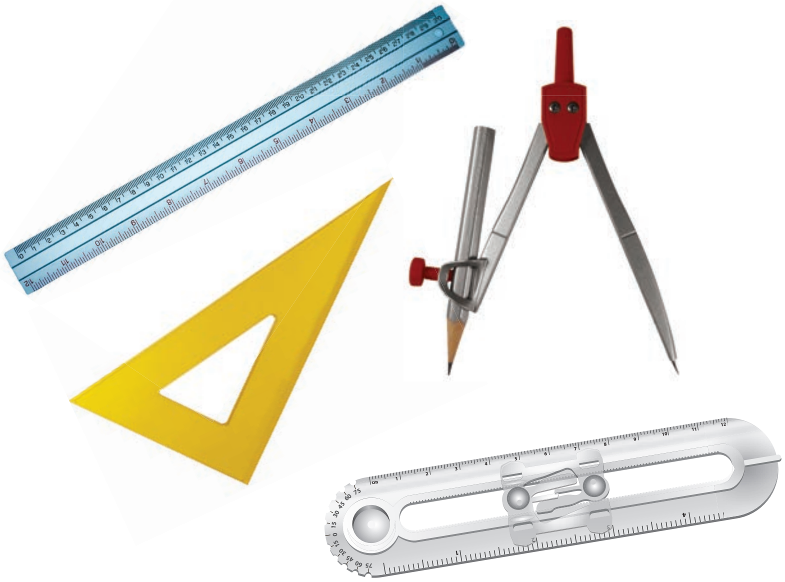
GO DIGITAL

Learning Target Measure and construct line segments.

Success Criteria
- I can measure a line segment.
- I can copy a line segment.
- I can explain and use the Segment Addition Postulate.

EXPLORE IT! Measuring and Copying Line Segments

Work with a partner. A *straightedge* is a tool that you can use to draw a straight line. An example of a straightedge is a ruler. A *compass* is a tool that you can use to draw circles and arcs, and copy segments. Choose from these tools to complete the following tasks.



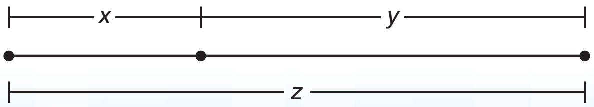
a. Find the length of the line segment.

b. MP **CHOOSE TOOLS** Make a copy of the line segment in part (a). Explain your process.

c. Draw a different line segment that has a length between 4 centimeters and 10 centimeters.

d. Make a copy of the line segment in part (c) using a different method than you used in part (b). Explain your process.

e. Find the lengths x, y, and z. What do you notice?



Math Practice

Make a Plan

How can you use a paper clip to compare the lengths of two different line segments?

GO DIGITAL

Using the Ruler Postulate

In geometry, a rule that is accepted without proof is called a **postulate** or an **axiom**. A rule that can be proved is called a *theorem*, as you will see later. Postulate 1.1 shows how to find the distance between two points on a line.

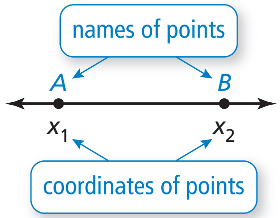
Vocabulary (AZ VOCAB)

postulate, *p. 12*
axiom, *p. 12*
coordinate, *p. 12*
distance between two points, *p. 12*
construction, *p. 13*
congruent segments, *p. 13*
between, *p. 14*

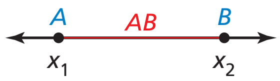
POSTULATE

1.1 Ruler Postulate

The points on a line can be matched one to one with the real numbers. The real number that corresponds to a point is the **coordinate** of the point.



The **distance** between points A and B, written as AB, is the absolute value of the difference of the coordinates of A and B.



$$AB = |x_2 - x_1|$$

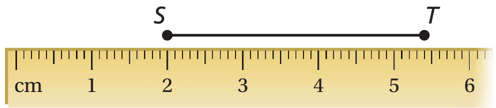
EXAMPLE 1 Using the Ruler Postulate

WATCH

Measure the length of $\overline{ST}$ to the nearest tenth of a centimeter.

SOLUTION

Align one mark of a metric ruler with S. Then estimate the coordinate of T. For example, when you align S with 2, T appears to align with 5.4.



$ST = |5.4 - 2| = 3.4$ Ruler Postulate

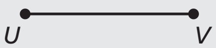
▶ So, the length of $\overline{ST}$ is about 3.4 centimeters.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Use a ruler to measure the length of the segment to the nearest $\frac{1}{8}$ inch.

1. 

2. 

3. 

4. 

5. WRITING Explain how $\overline{XY}$ and XY are different.

Constructing and Comparing Congruent Segments

GO DIGITAL

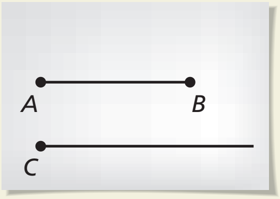
A **construction** is a geometric drawing that uses a limited set of tools, usually a *compass* and *straightedge*.

CONSTRUCTION **Copying a Segment** WATCH

Use a compass and straightedge to construct a line segment that has the same length as $\overline{AB}$.

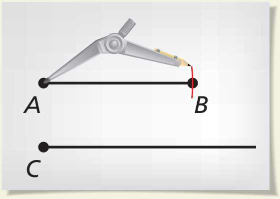
SOLUTION

Step 1



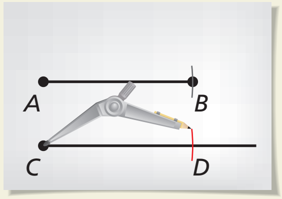
Draw a segment Use a straightedge to draw a segment longer than $\overline{AB}$. Label point C on the new segment.

Step 2



Measure length Set your compass at the length of $\overline{AB}$.

Step 3

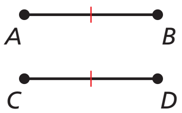


Copy length Place the compass at C. Mark point D on the new segment. So, $\overline{CD}$ has the same length as $\overline{AB}$.

KEY IDEA

Congruent Segments

Line segments that have the same length are called **congruent segments**. You can say "the length of $\overline{AB}$ is equal to the length of $\overline{CD}$," or you can say "$\overline{AB}$ *is congruent to* $\overline{CD}$." The symbol $\cong$ means "is congruent to."



Lengths are equal.

$AB = CD$

"is equal to"

Segments are congruent.

$\overline{AB} \cong \overline{CD}$

"is congruent to"

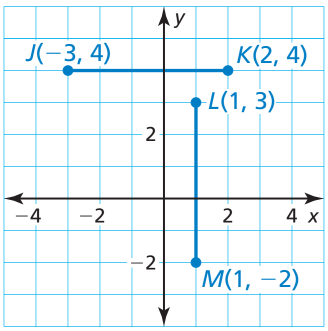
READING

In the diagram, the red tick marks indicate $\overline{AB} \cong \overline{CD}$. When there is more than one pair of congruent segments, use multiple tick marks.

EXAMPLE 2 **Comparing Segments for Congruence** WATCH

Plot $J(-3, 4)$, $K(2, 4)$, $L(1, 3)$, and $M(1, -2)$ in a coordinate plane. Then determine whether $\overline{JK}$ and $\overline{LM}$ are congruent.

SOLUTION



Plot the points, as shown. To find the length of a horizontal segment, find the absolute value of the difference of the x-coordinates of the endpoints.

$JK = |2 - (-3)| = 5$ Ruler Postulate

To find the length of a vertical segment, find the absolute value of the difference of the y-coordinates of the endpoints.

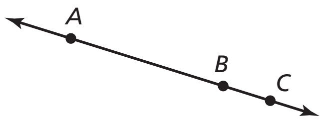
$LM = |-2 - 3| = 5$ Ruler Postulate

▶ $\overline{JK}$ and $\overline{LM}$ have the same length. So, $\overline{JK} \cong \overline{LM}$.

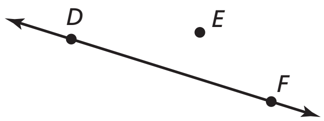
Using the Segment Addition Postulate

GO DIGITAL

When three points are collinear, you can say that one point is **between** the other two.



Point *B* is between points *A* and *C*.



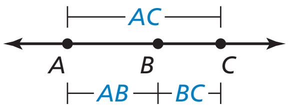
Point *E* is not between points *D* and *F*.

POSTULATE

1.2 Segment Addition Postulate

If B is between A and C, then $AB + BC = AC$.

If $AB + BC = AC$, then B is between A and C.

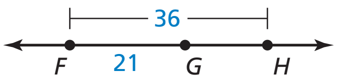


EXAMPLE 3 Using the Segment Addition Postulate

WATCH

a. Find DF.

b. Find GH.



SOLUTION

a. Use the Segment Addition Postulate to write an equation. Then solve the equation to find DF.

$DF = DE + EF$	Segment Addition Postulate
$DF = 23 + 35$	Substitute 23 for *DE* and 35 for *EF*.
$DF = 58$	Add.

b. Use the Segment Addition Postulate to write an equation. Then solve the equation to find GH.

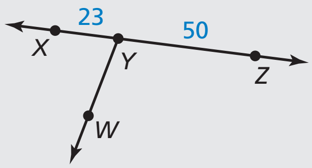
$FH = FG + GH$	Segment Addition Postulate
$36 = 21 + GH$	Substitute 36 for *FH* and 21 for *FG*.
$15 = GH$	Subtract 21 from each side.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

6. Plot $A(-2, 4)$, $B(3, 4)$, $C(0, 2)$, and $D(0, -2)$ in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.

Use the diagram.



7. Find XZ.

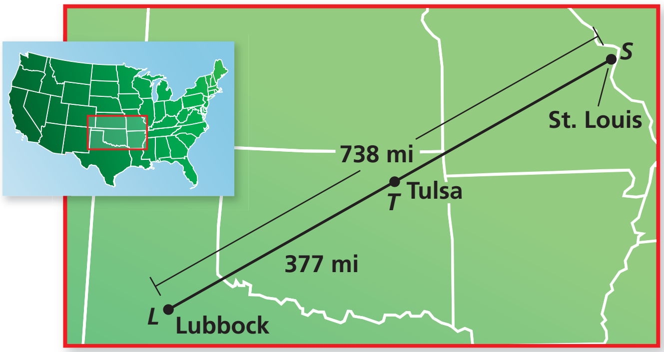
8. In the diagram, $WY = 30$. Can you use the Segment Addition Postulate to find the distance between points W and Z? Explain your reasoning.

GO DIGITAL

EXAMPLE 4 Modeling Real Life

WATCH

The cities shown on the map lie approximately in a straight line. Find the distance from Tulsa, Oklahoma, to St. Louis, Missouri.



SOLUTION

1. **Understand the Problem** You know that the three cities are approximately collinear. The map shows the distances from Lubbock to St. Louis and from Lubbock to Tulsa. You need to find the distance from Tulsa to St. Louis.
2. **Make a Plan** Use the Segment Addition Postulate to find the distance from Tulsa to St. Louis.
3. **Solve and Check** Use the Segment Addition Postulate to write an equation. Then solve the equation to find TS.

$LS = LT + TS$	Segment Addition Postulate
$738 = 377 + TS$	Substitute 738 for LS and 377 for LT.
$361 = TS$	Subtract 377 from each side.

▶ So, the distance from Tulsa to St. Louis is about 361 miles.

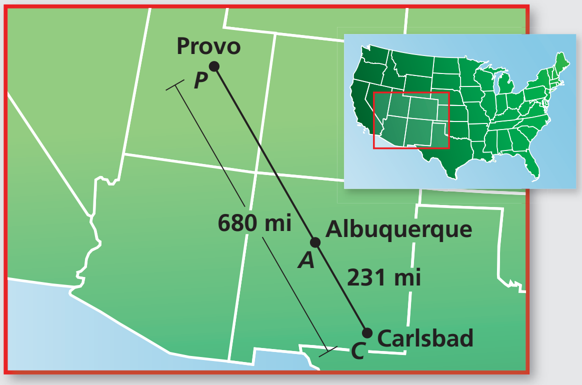
Check The distance from Lubbock to St. Louis is 738 miles. By the Segment Addition Postulate, the distance from Lubbock to Tulsa plus the distance from Tulsa to St. Louis should equal 738 miles.

$$377 + 361 = 738 \checkmark$$

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

9. The cities shown on the map lie approximately in a straight line. Find the distance from Albuquerque, New Mexico, to Provo, Utah.



1.2 Practice WITH CalcChat® AND CalcView®

GO DIGITAL

In Exercises 1–4, use a ruler to measure the length of the segment to the nearest tenth of a centimeter.
▶ *Example 1*

1.

2.

3.

4.

CONSTRUCTION **In Exercises 5 and 6, use a compass and straightedge to construct a copy of the segment.**

5. Copy the segment in Exercise 3.

6. Copy the segment in Exercise 4.

In Exercises 7–12, plot the points in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.
▶ *Example 2*

7. $A(-4, 5)$, $B(-4, 8)$, $C(2, -3)$, $D(2, 0)$

8. $A(6, -1)$, $B(1, -1)$, $C(2, -3)$, $D(4, -3)$

9. $A(8, 3)$, $B(-1, 3)$, $C(5, 10)$, $D(5, 3)$

10. $A(6, -8)$, $B(6, 1)$, $C(7, -2)$, $D(-2, -2)$

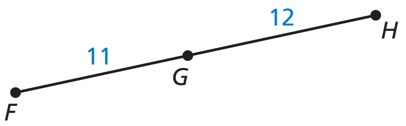
11. $A(-5, 6)$, $B(-5, -1)$, $C(-4, 3)$, $D(3, 3)$

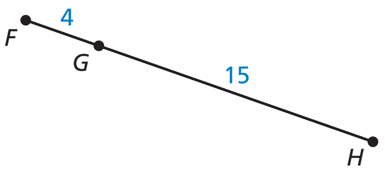
12. $A(10, -4)$, $B(3, -4)$, $C(-1, 2)$, $D(-1, 5)$

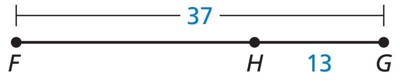
In Exercises 13–20, find *FH*. ▶ *Example 3*

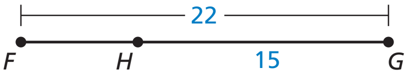
13. 

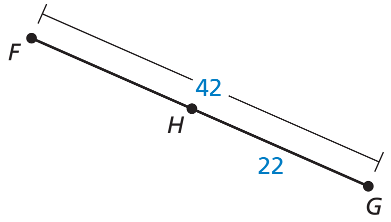
14. 

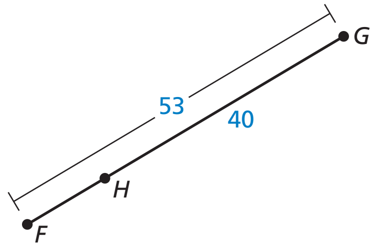
15. 

16. 

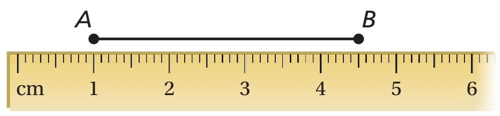
17. 

18. 

19. 

20. 

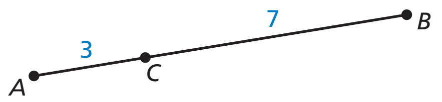
ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in finding the length of $\overline{AB}$.**



21. ✗ $AB = 1 - 4.5 = -3.5$

22. ✗ $AB = |1 + 4.5| = 5.5$

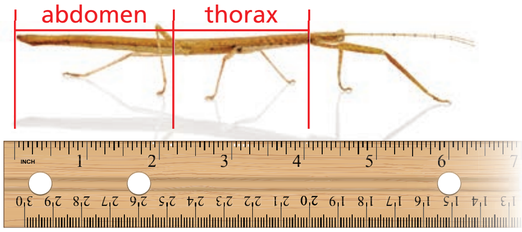
23. **COLLEGE PREP** Which expression does *not* equal 10?



Ⓐ $AC + CB$ Ⓑ $BA - CA$

Ⓒ AB Ⓓ $CA + BC$

24. **MP PRECISION** The diagram shows an insect called a walking stick. Use the ruler to estimate the length of the abdomen and the length of the thorax to the nearest $\frac{1}{4}$ inch. How much longer is the walking stick's abdomen than its thorax? How many times longer is its abdomen than its thorax?

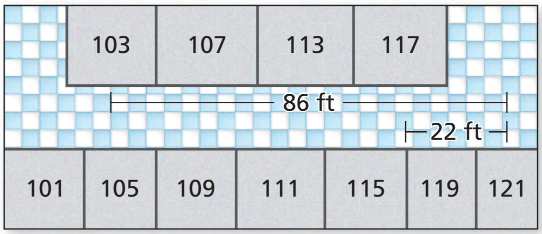


25. **MODELING REAL LIFE** In 2003, a remote-controlled model airplane became the first ever to fly nonstop across the Atlantic Ocean. The map shows the airplane's position at three different points during its flight. Point A represents Cape Spear, Newfoundland, point B represents the approximate position after 1 day, and point C represents Mannin Bay, Ireland. The airplane left from Cape Spear and landed in Mannin Bay. *Example 4*

a. Find the total distance the model airplane flew.

b. The flight lasted nearly 38 hours. Estimate the airplane's average speed in miles per hour.

26. **MODELING REAL LIFE** You walk in a straight line from Room 103 to Room 117 at a speed of 4.4 feet per second.

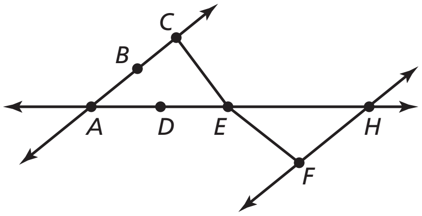


a. How far do you walk?

b. How long does it take you to get to Room 117?

c. Why might it actually take you longer than the time in part (b)?

27. **MP STRUCTURE** Determine whether each statement is *true* or *false*. Explain your reasoning.



a. B is between A and C.

b. C is between B and E.

c. D is between A and H.

d. E is between C and F.

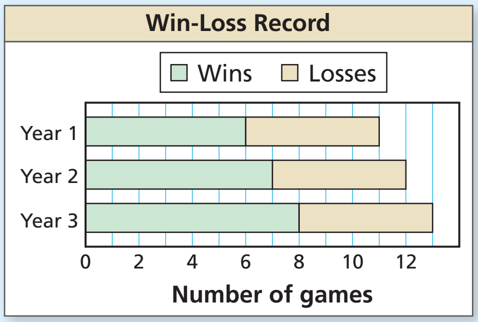
28. **CONNECTING CONCEPTS** Point S is between points R and T on $\overline{RT}$. Use the information to write an equation in terms of x. Then solve the equation and find RS, ST, and RT.

a. $RS = 2x + 10$
$ST = x - 4$
$RT = 21$

b. $RS = 4x - 9$
$ST = 19$
$RT = 8x - 14$

29. **MAKING AN ARGUMENT** Your friend says that when measuring with a ruler, you must always line up objects at the zero on the ruler. Is your friend correct? Explain your reasoning.

30. **HOW DO YOU SEE IT?**
The bar graph shows the win-loss record for a lacrosse team over a period of three years. Explain how you can apply the Ruler Postulate and the Segment Addition Postulate when interpreting a stacked bar graph like the one shown.

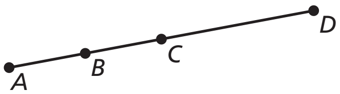


31. **MP REASONING** The round-trip distance between City X and City Y is 647 miles. A national park is between City X and City Y, and is 27 miles from City X. Find the round-trip distance between the national park and City Y. Justify your answer.

GO DIGITAL

32. **ABSTRACT REASONING** The points (a, b) and (c, b) form a segment, and the points (d, e) and (d, f) form a segment. The segments are congruent. Write an equation that represents the relationship among the variables. Are any of the variables *not* used in the equation? Explain.

33. **CONNECTING CONCEPTS** In the diagram, $\overline{AB} \cong \overline{BC}$, $\overline{AC} \cong \overline{CD}$, and $AD = 12$. Find the lengths of all segments in the diagram. You choose one of the segments at random. What is the probability that the length of the segment is greater than 3? Explain your reasoning.



34. **CRITICAL THINKING** Points A, B, and C lie on a line where $AB = 35$ and $AC = 93$. What are the possible values of BC?

GO DIGITAL

35. **DIG DEEPER** Is it possible to use the Segment Addition Postulate to show that $FB > CB$? that $AC > DB$? Explain your reasoning.

36. **THOUGHT PROVOKING**
Is it possible to design a table where no two legs have the same length? Assume that the endpoints of the legs (that are not attached to the table) must all lie in the same plane. Include a diagram with your answer.

REVIEW & REFRESH

WATCH

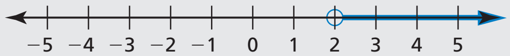
In Exercises 37–40, solve the equation.

37. $3 + y = 12$ **38.** $-5x = 10$

39. $5x + 7 = 9x - 17$ **40.** $\dfrac{-5 + x}{2} = -9$

41. Sketch plane P and $\overleftrightarrow{YZ}$ intersecting at point Z.

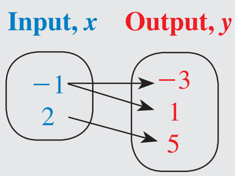
42. Write an inequality that represents the graph.



In Exercises 43 and 44, use intercepts to graph the linear equation. Label the points corresponding to the intercepts.

43. $4x + 3y = 24$ **44.** $-2x + 4y = -16$

45. Determine whether the relation is a function. Explain.



In Exercises 46–49, solve the inequality. Graph the solution.

46. $x - 6 \leq 13$ **47.** $-3t > 15$

48. $5 - \dfrac{c}{3} < 12$

49. $6 - v < 8$ *or* $-4v \geq 40$

In Exercises 50 and 51, graph the function. Identify the asymptote. Find the domain and range of *f*.

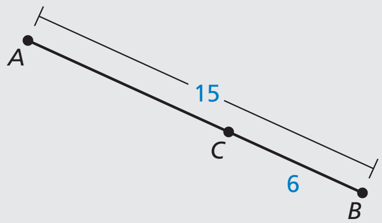
50. $f(x) = 2^x - 3$ **51.** $f(x) = 3(0.5)^{x-1}$

52. Is there a correlation between amusement park attendance and the wait times for rides? If so, is there a causal relationship? Explain your reasoning.

In Exercises 53 and 54, find *AC*.

53.

54.



55. **MODELING REAL LIFE** A football team scores a total of 7 touchdowns and field goals in a game. The team scores an extra point with each touchdown, so each touchdown is worth 7 points and each field goal is worth 3 points. The team scores a total of 41 points. How many touchdowns does the team score? How many field goals?

In Exercises 56 and 57, write an equation in slope-intercept form of the line that passes through the given points.

56. $(0, 3), \left(\frac{1}{2}, 0\right)$ **57.** $(-8, -8), (12, -3)$

1.3 Using Midpoint and Distance Formulas

GO DIGITAL

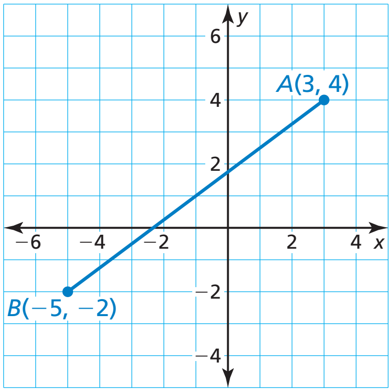
Learning Target Find midpoints and lengths of segments.

Success Criteria
- I can find lengths of segments.
- I can construct a segment bisector.
- I can find the midpoint of a segment.

EXPLORE IT! Finding Midpoints of Line Segments

Work with a partner.

a. Plot any two points A and B. Then graph $\overline{AB}$. Identify the point M on $\overline{AB}$ that is halfway between points A and B, called the *midpoint* of $\overline{AB}$. Explain how you found the midpoint.



Math Practice

Use a Diagram
Draw a right triangle with hypotenuse $\overline{AB}$. How can you find the length of $\overline{AB}$?

b. Repeat part (a) five times and complete the table.

Coordinates of A	Coordinates of B	Coordinates of M

c. Compare the x-coordinates of A, B, and M. Compare the y-coordinates of A, B, and M. How are the coordinates of the midpoint M related to the coordinates of A and B?

GO DIGITAL

Vocabulary VOCAB

midpoint, *p. 20*
segment bisector, *p. 20*

READING

The word *bisect* means "to cut into two equal parts."

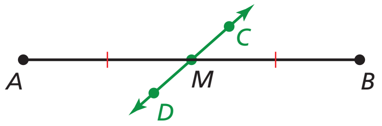
KEY IDEAS

Midpoints and Segment Bisectors

The **midpoint** of a segment is the point that divides the segment into two congruent segments.

M is the midpoint of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.

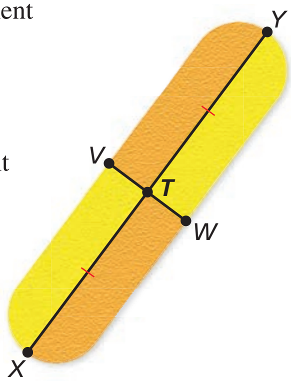


$\overleftrightarrow{CD}$ is a segment bisector of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and $AM = MB$.

EXAMPLE 1 Finding Segment Lengths

WATCH INFO

In the skateboard design, $XT = 39.9$ cm. Identify the segment bisector of $\overline{XY}$. Then find XY.



SOLUTION

The design shows that $\overline{XT} \cong \overline{TY}$. So, point T is the midpoint of $\overline{XY}$ and $XT = TY = 39.9$ cm. Because $\overline{VW}$ intersects $\overline{XY}$ at its midpoint T, $\overline{VW}$ bisects $\overline{XY}$. Find XY.

$XY = XT + TY$	Segment Addition Postulate
$= 39.9 + 39.9$	Substitute.
$= 79.8$	Add.

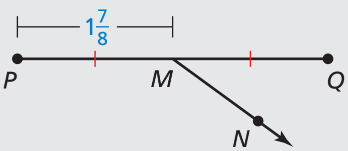
▶ $\overline{VW}$ is the segment bisector of $\overline{XY}$, and XY is 79.8 centimeters.

SELF-ASSESSMENT

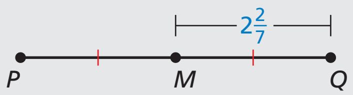
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Identify the segment bisector of $\overline{PQ}$. Then find PQ.

1.

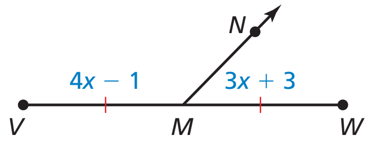


2.



3. VOCABULARY If a point, ray, line, line segment, or plane intersects a segment at its midpoint, then what does it do to the segment?

EXAMPLE 2 Using Algebra with Segment Lengths



Identify the segment bisector of $\overline{VW}$. Then find VM.

WATCH

SOLUTION

The figure shows that $\overline{VM} \cong \overline{MW}$. So, point M is the midpoint of $\overline{VW}$ and $VM = MW$. Because $\overrightarrow{MN}$ intersects $\overline{VW}$ at its midpoint M, $\overrightarrow{MN}$ bisects $\overline{VW}$. Find VM.

Step 1 Write and solve an equation to find VM.

$VM = MW$	Write the equation.
$4x - 1 = 3x + 3$	Substitute.
$x - 1 = 3$	Subtract $3x$ from each side.
$x = 4$	Add 1 to each side.

Step 2 Evaluate the expression for VM when $x = 4$.

$$VM = 4x - 1 = 4(4) - 1 = 15$$

▶ $\overrightarrow{MN}$ is the segment bisector of $\overline{VW}$, and VM is 15.

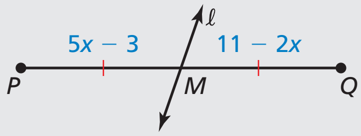
Check Because $VM = MW$, the length of $\overline{MW}$ should be 15.

$$MW = 3x + 3$$
$$= 3(4) + 3$$
$$= 15 \checkmark$$

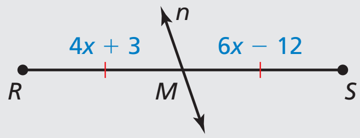
SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

4. Identify the segment bisector of $\overline{PQ}$. Then find MQ.



5. Identify the segment bisector of $\overline{RS}$. Then find RS.



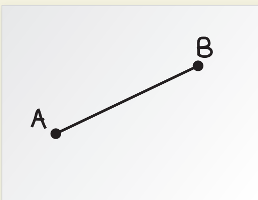
CONSTRUCTION Bisecting a Segment

WATCH

Construct a segment bisector of $\overline{AB}$ by paper folding. Then label the midpoint M of $\overline{AB}$.

SOLUTION

Step 1

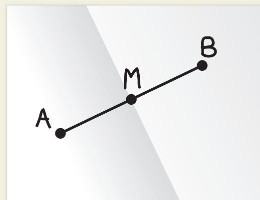


Draw a segment
Use a straightedge to draw $\overline{AB}$ on a piece of paper.

Step 2

Fold the paper
Fold the paper so that B is on top of A.

Step 3



Label the midpoint
Label point M. Compare AM, MB, and AB.

$$AM = MB = \tfrac{1}{2}AB$$

Using the Midpoint Formula

GO DIGITAL

You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

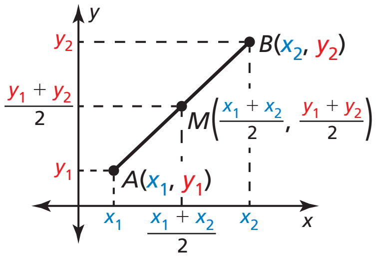
KEY IDEA

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



EXAMPLE 3 Using the Midpoint Formula

WATCH

a. The endpoints of $\overline{RS}$ are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint M.

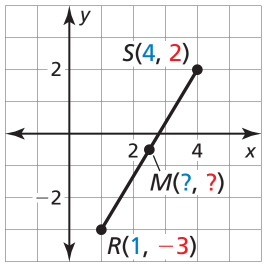
b. The midpoint of $\overline{JK}$ is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint K.

SOLUTION

a. Use the Midpoint Formula.

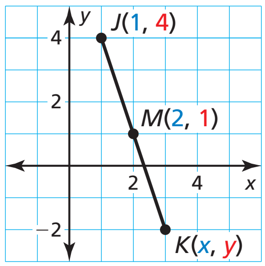
$$M\left(\frac{1 + 4}{2}, \frac{-3 + 2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

▶ The coordinates of the midpoint M are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.



b. Let (x, y) be the coordinates of endpoint K. Use the Midpoint Formula.

Step 1 Find x.	**Step 2** Find y.
$\frac{1 + x}{2} = 2$	$\frac{4 + y}{2} = 1$
$1 + x = 4$	$4 + y = 2$
$x = 3$	$y = -2$



▶ The coordinates of endpoint K are $(3, -2)$.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

The endpoints of $\overline{AB}$ are given. Find the coordinates of the midpoint M.

6. $A(1, 2)$ and $B(7, 8)$

7. $A(-4, 3)$ and $B(-6, 5)$

The midpoint M and one endpoint of $\overline{TU}$ are given. Find the coordinates of the other endpoint.

8. $T(1, 1)$ and $M(2, 4)$

9. $U(4, 4)$ and $M(-1, -2)$

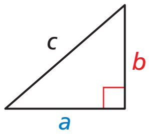
Using the Distance Formula

GO DIGITAL

You can use the Distance Formula to find the distance between two points in a coordinate plane. You can derive the Distance Formula from the *Pythagorean Theorem*, which you will see again when you work with right triangles.

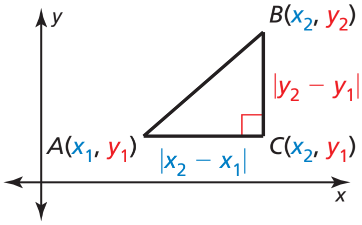
Pythagorean Theorem

$c^2 = a^2 + b^2$



Distance Formula

$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$



READING

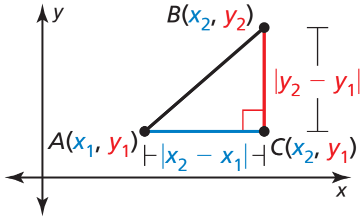
The red mark at the corner of the triangle that makes a right angle indicates a right triangle.

KEY IDEA

The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



EXAMPLE 4 **Using the Distance Formula**

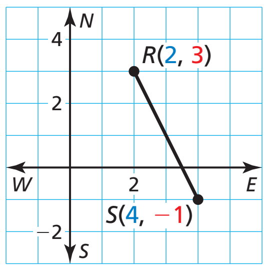
WATCH

Your school is 4 miles east and 1 mile south of your apartment. A recycling center, where your class is going on a field trip, is 2 miles east and 3 miles north of your apartment. Estimate the distance between the recycling center and your school.

SOLUTION

You can model the situation using a coordinate plane with your apartment at the origin (0, 0). The coordinates of the recycling center and the school are $R(2, 3)$ and $S(4, -1)$, respectively. Use the Distance Formula. Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, -1)$.

$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{(4 - 2)^2 + (-1 - 3)^2}$	Substitute.
$= \sqrt{2^2 + (-4)^2}$	Subtract.
$= \sqrt{4 + 16}$	Evaluate powers.
$= \sqrt{20}$	Add.
≈ 4.5	Use technology.



READING

The symbol ≈ means "is approximately equal to."

So, the distance between the recycling center and your school is about 4.5 miles.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

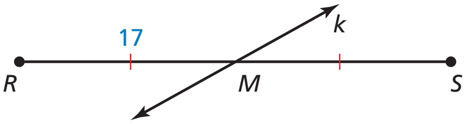
10. In Example 4, a park is 3 miles east and 4 miles south of your apartment. Estimate the distance between the park and your school.

1.3 Practice WITH CalcChat® AND CalcView®

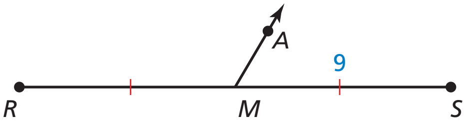
GO DIGITAL

In Exercises 1–4, identify the segment bisector of $\overline{RS}$. Then find RS. ▷ *Example 1*

1.



2.



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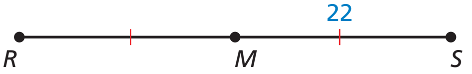
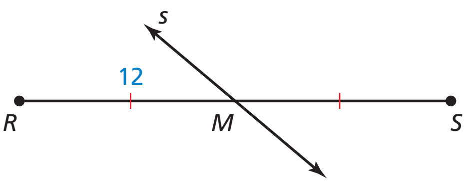
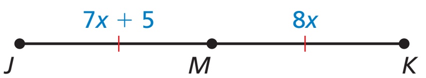


4.



In Exercises 5 and 6, identify the segment bisector of $\overline{JK}$. Then find JM. ▷ *Example 2*

5.



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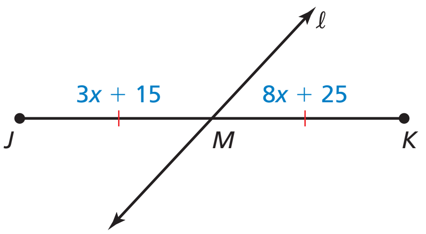
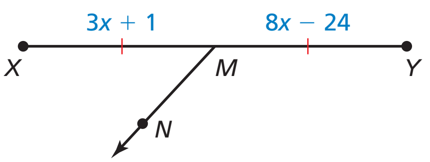
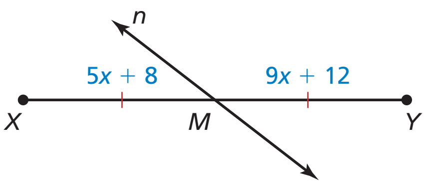


In Exercises 7 and 8, identify the segment bisector of $\overline{XY}$. Then find XY. ▷ *Example 2*

7.



8.

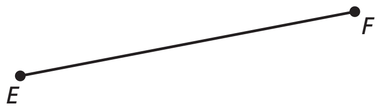


CONSTRUCTION In Exercises 9–12, copy the segment and construct a segment bisector by paper folding. Then label the midpoint M.

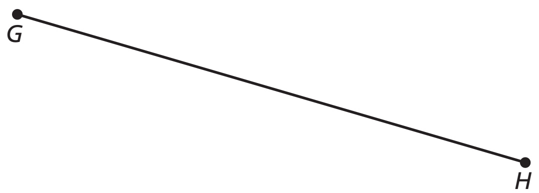
9.

10.

11.



12.



In Exercises 13–16, the endpoints of $\overline{CD}$ are given. Find the coordinates of the midpoint M. ▷ *Example 3*

13. $C(3, -5)$ and $D(7, 9)$

14. $C(-4, 7)$ and $D(0, -3)$

15. $C(-2, 0)$ and $D(4, 9)$

16. $C(-8, -6)$ and $D(-4, 10)$

In Exercises 17–20, the midpoint M and one endpoint of $\overline{GH}$ are given. Find the coordinates of the other endpoint. ▷ *Example 3*

17. $G(5, -6)$ and $M(4, 3)$

18. $H(-3, 7)$ and $M(-2, 5)$

19. $H(-2, 9)$ and $M(8, 0)$

20. $G(-4, 1)$ and $M\left(-\frac{13}{2}, -6\right)$

In Exercises 21–28, find the distance between the two points. ▷ *Example 4*

21. $A(13, 2)$ and $B(7, 10)$

22. $C(-6, 5)$ and $D(-3, 1)$

23. $E(3, 7)$ and $F(6, 5)$

24. $G(-5, 4)$ and $H(2, 6)$

25. $J(-8, 0)$ and $K(1, 4)$

26. $L(7, -1)$ and $M(-2, 4)$

27. $R(0, 1)$ and $S(6, 3.5)$

28. $T(13, 1.6)$ and $V(5.4, 3.7)$

GO DIGITAL

ERROR ANALYSIS **In Exercises 29 and 30, describe and correct the error in finding the distance between $A(6, 2)$ and $B(1, -4)$.**

29. ✗

$$AB = (6-1)^2 + [2-(-4)]^2$$
$$= 5^2 + 6^2$$
$$= 25 + 36$$
$$= 61$$

30. ✗

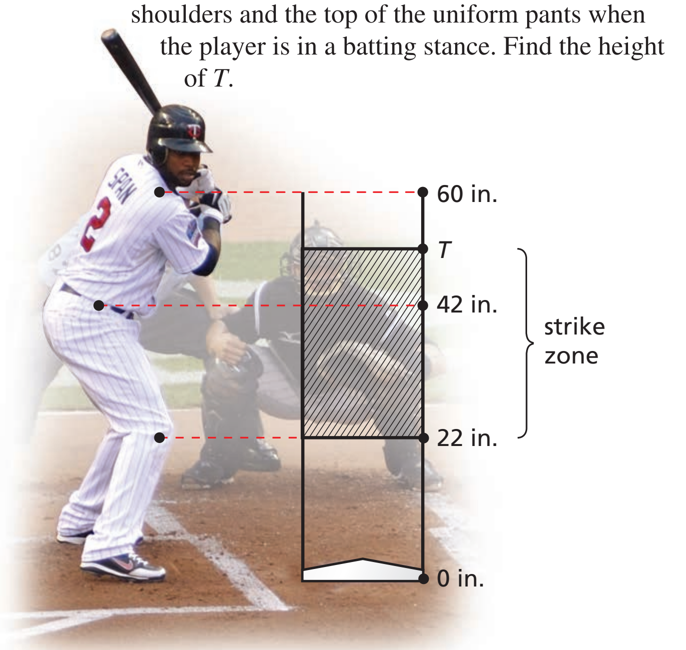
$$AB = \sqrt{(6-2)^2 + [1-(-4)]^2}$$
$$= \sqrt{4^2 + 5^2}$$
$$= \sqrt{16 + 25}$$
$$= \sqrt{41}$$

In Exercises 31 and 32, the endpoints of two segments are given. Find the length of each segment. Tell whether the segments are congruent. If they are not congruent, tell which segment is longer.

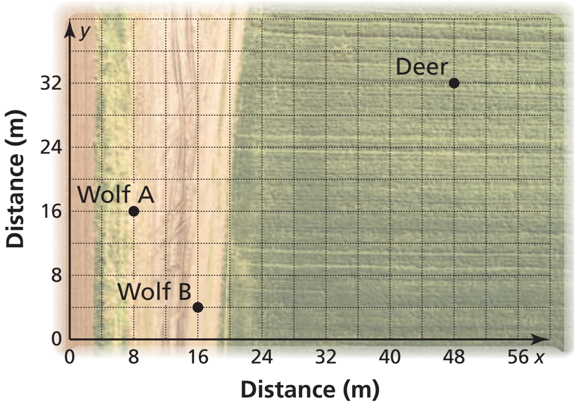
31. $\overline{AB}$: $A(0, 2)$, $B(-3, 8)$ and $\overline{CD}$: $C(-2, 2)$, $D(0, -4)$

32. $\overline{EF}$: $E(1, 4)$, $F(5, 1)$ and $\overline{GH}$: $G(-3, 1)$, $H(1, 6)$

33. **MODELING REAL LIFE** In baseball, the strike zone is the region a baseball needs to pass through for the umpire to declare it a strike when the batter does not swing. The bottom of the strike zone is a horizontal plane passing through a point just below the kneecap. The top of the strike zone is a horizontal plane passing through the midpoint of the top of the batter's shoulders and the top of the uniform pants when the player is in a batting stance. Find the height of *T*.



34. **MODELING REAL LIFE** Two wolves spot a deer in a field. The positions of the animals are shown. Which wolf is closer to the deer?

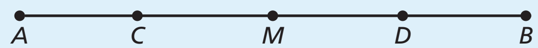


35. **MODELING REAL LIFE** A theater is 3 miles east and 1 mile north of a bus stop. A museum is 4 miles west and 3 miles south of the bus stop. Estimate the distance between the theater and the museum.

36. **MODELING REAL LIFE** Your school is 20 blocks east and 12 blocks south of your home. The mall, where you plan to go after school, is 7 blocks west and 10 blocks north of your home. One block is 0.1 mile. Estimate the distance in miles between your school and the mall.

37. **MAKING AN ARGUMENT** Your friend claims there is an easier way to find the length of a segment than using the Distance Formula when the *x*-coordinates of the endpoints are equal. He claims all you have to do is subtract the *y*-coordinates. Do you agree with his statement? Explain your reasoning.

38. **HOW DO YOU SEE IT?**
$\overline{AB}$ contains midpoint *M* and points *C* and *D*, as shown. Compare the lengths. If you cannot draw a conclusion, write *impossible to tell*. Explain your reasoning.

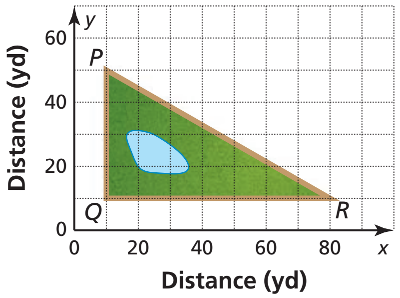


a. *AM* and *MB* **b.** *AC* and *MB*

c. *CM* and *MD* **d.** *MB* and *DB*

39. **CRITICAL THINKING** The endpoints of a segment are located at (a, c) and (b, c). Find the coordinates of the midpoint and the length of the segment in terms of a, b, and c.

40. **MP PROBLEM SOLVING** A new bridge is constructed in the triangular park shown. The bridge spans from point Q to the midpoint M of $\overline{PR}$. A person jogs from P to Q to M to R to Q and back to P at an average speed of 150 yards per minute. About how many minutes does it take? Explain your reasoning.



41. **ANALYZING RELATIONSHIPS** The length of $\overline{XY}$ is 24 centimeters. The midpoint of $\overline{XY}$ is M, and point C lies on $\overline{XM}$ so that XC is $\frac{2}{3}$ of XM. Point D lies on $\overline{MY}$ so that MD is $\frac{3}{4}$ of MY. What is the length of $\overline{CD}$?

GO DIGITAL

42. **THOUGHT PROVOKING**
The distance between $K(1, -5)$ and a point L with integer coordinates is $\sqrt{58}$ units. Find all the possible coordinates of point L.

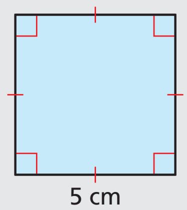
43. **DIG DEEPER** The endpoints of $\overline{AB}$ are $A(2x, y - 1)$ and $B(y + 3, 3x + 1)$. The midpoint of $\overline{AB}$ is $M\left(-\frac{7}{2}, -8\right)$. What is the length of $\overline{AB}$?

REVIEW & REFRESH

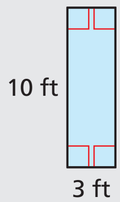
WATCH

In Exercises 44–47, find the perimeter and area of the figure.

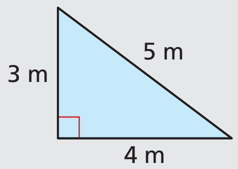
44.



45.



46.



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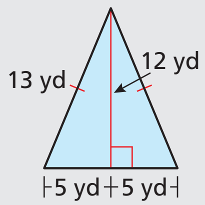


In Exercises 48–51, solve the inequality. Graph the solution.

48. $a + 18 < 7$

49. $y - 5 \geq 8$

50. $-3x > 24$

51. $\frac{z}{4} \leq 12$

52. The endpoints of $\overline{YZ}$ are $Y(1, -6)$ and $Z(-2, 8)$. Find the coordinates of the midpoint M. Then find YZ.

53. Solve the literal equation $5x + 15y = -30$ for y.

54. Find the average rate of change of $f(x) = 3^x$ from $x = 1$ to $x = 3$.

In Exercises 55–58, factor the polynomial.

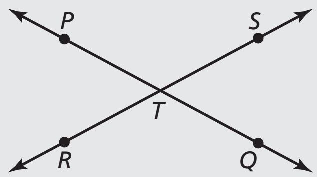
55. $3x^2 - 36x$

56. $n^2 + 3n - 70$

57. $121p^2 - 100$

58. $15y^2 + 4y - 4$

59. Name two pairs of opposite rays in the diagram.



In Exercises 60 and 61, simplify the expression. Write your answer using only positive exponents.

60. $\frac{b^4 \cdot b^{-2}}{b^{10}}$

61. $\left(\frac{2}{5t^4}\right)^{-3}$

62. Plot $A(-3, 3)$, $B(1, 3)$, $C(3, 2)$, and $D(3, -2)$ in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.

63. **MODELING REAL LIFE** The function $p(x) = 80 - 2x$ represents the number of points earned on a test with x incorrect answers.

a. How many points are earned with 2 incorrect answers?

b. How many incorrect answers are there when 68 points are earned?

64. Convert 320 fluid ounces to gallons.

1.4 Perimeter and Area in the Coordinate Plane

Learning Target Find perimeters and areas of polygons in the coordinate plane.

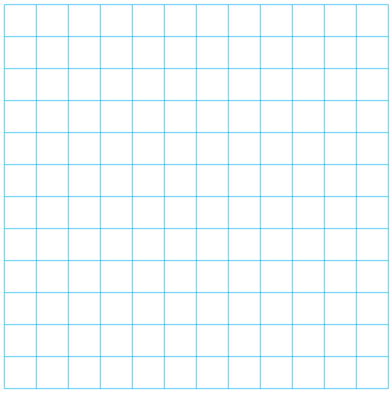
Success Criteria

- I can classify and describe polygons.
- I can find perimeters of polygons in the coordinate plane.
- I can find areas of polygons in the coordinate plane.

EXPLORE IT! Finding the Perimeter and Area of a Quadrilateral

Work with a partner.

a. Use a piece of graph paper to draw a quadrilateral *ABCD* in a coordinate plane. At most two sides of your quadrilateral can be horizontal or vertical. Plot and label the vertices of *ABCD*.

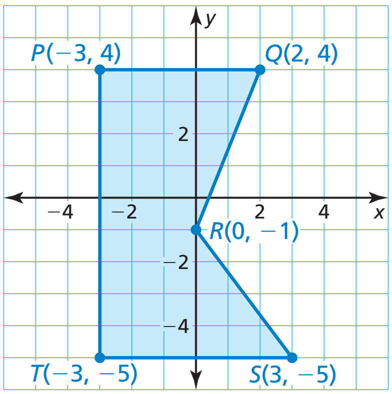


b. Make several observations about quadrilateral *ABCD*. Can you use any other names to classify your quadrilateral? Explain.

c. Explain how you can find the perimeter of quadrilateral *ABCD*. Then find the perimeter. Compare your method with those of your classmates.

d. Explain how you can find the area of quadrilateral *ABCD*. Then find the area. Compare your method with those of your classmates.

e. Use the methods from parts (c) and (d) to find the perimeter and area of the polygon below. Explain your reasoning.



Math Practice

Look for Structure

In part (d), how might it be helpful to visualize a polygon as being composed of one or more smaller polygons?

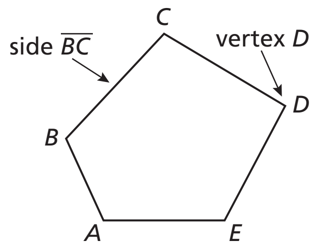
GO DIGITAL

Classifying Polygons

KEY IDEA

Polygons

In geometry, a figure that lies in a plane is called a plane figure. Recall that a *polygon* is a closed plane figure formed by three or more line segments called *sides*. Each side intersects exactly two sides, one at each *vertex*, so that no two sides with a common vertex are collinear.



polygon *ABCDE*

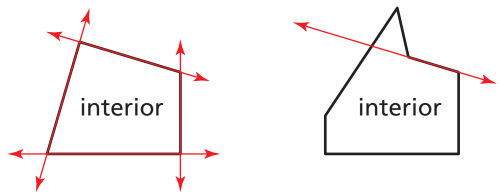
READING

You can name a polygon by listing the vertices in consecutive order.

Number of sides	Type of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	n-gon

The number of sides determines the type of polygon, as shown in the table. You can also name a polygon using the term n-gon, where n is the number of sides. For instance, a 14-gon is a polygon with 14 sides.

A polygon is *convex* when no line that contains a side of the polygon contains a point in the interior of the polygon. A polygon that is not convex is *concave*.



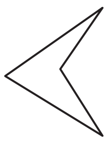
convex polygon   concave polygon

EXAMPLE 1 Classifying Polygons

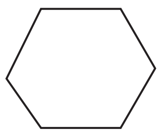
WATCH

Classify each polygon by the number of sides. Tell whether it is *convex* or *concave*.

a.



b.



SOLUTION

a. The polygon has four sides. So, it is a quadrilateral. The polygon is concave.

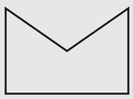
b. The polygon has six sides. So, it is a hexagon. The polygon is convex.

SELF-ASSESSMENT

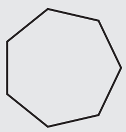
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Classify the polygon by the number of sides. Tell whether it is *convex* or *concave*.

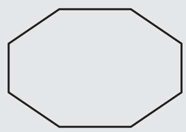
1.



2.



3.



4. MP **REASONING** Can you draw a concave triangle? If so, draw one. If not, explain why not.

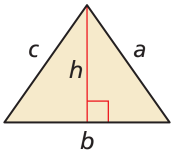
Finding Perimeter and Area in the Coordinate Plane

GO DIGITAL

You can use the formulas below and the Distance Formula to find perimeters and areas of polygons in the coordinate plane.

Perimeter and Area

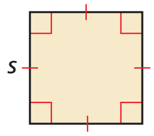
Triangle



$P = a + b + c$

$A = \frac{1}{2}bh$

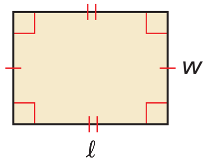
Square



$P = 4s$

$A = s^2$

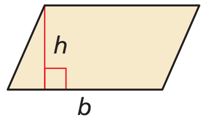
Rectangle



$P = 2\ell + 2w$

$A = \ell w$

Parallelogram



$A = bh$

READING

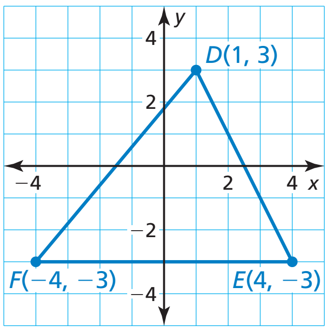
You can read the notation $\triangle DEF$ as "triangle *D E F*."

EXAMPLE 2 **Finding Perimeter in the Coordinate Plane**

WATCH

Find the perimeter of $\triangle DEF$ with vertices $D(1, 3)$, $E(4, -3)$, and $F(-4, -3)$.

SOLUTION



Step 1 Draw the triangle in a coordinate plane by plotting the vertices and connecting them.

Step 2 Find the length of each side.

$\overline{DE}$ Let $(x_1, y_1) = (1, 3)$ and $(x_2, y_2) = (4, -3)$.

$DE = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Distance Formula

$= \sqrt{(4 - 1)^2 + (-3 - 3)^2}$ — Substitute.

$= \sqrt{3^2 + (-6)^2}$ — Subtract.

$= \sqrt{45}$ — Simplify.

$\overline{EF}$ $EF = |-4 - 4| = |-8| = 8$ — Ruler Postulate

$\overline{FD}$ Let $(x_1, y_1) = (-4, -3)$ and $(x_2, y_2) = (1, 3)$.

$FD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ — Distance Formula

$= \sqrt{[1 - (-4)]^2 + [3 - (-3)]^2}$ — Substitute.

$= \sqrt{5^2 + 6^2}$ — Subtract.

$= \sqrt{61}$ — Simplify.

REMEMBER

Perimeter has linear units, such as feet or meters. Area has square units, such as square feet or square meters.

Step 3 Find the sum of the side lengths.

$$DE + EF + FD = \sqrt{45} + 8 + \sqrt{61} \approx 22.52 \text{ units}$$

▶ So, the perimeter of $\triangle DEF$ is about 22.52 units.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the perimeter of the polygon with the given vertices.

5. $G(-3, 2)$, $H(2, 2)$, $J(-1, -3)$

6. $Q(-4, -1)$, $R(1, 4)$, $S(4, 1)$, $T(-1, -4)$

EXAMPLE 3 Finding Area in the Coordinate Plane

GO DIGITAL

WATCH

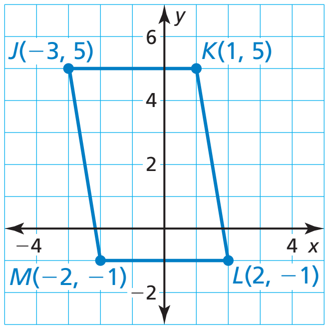
READING

You can read the notation ▱*JKLM* as "parallelogram *J K L M*."

Find the area of ▱$JKLM$ with vertices $J(-3, 5)$, $K(1, 5)$, $L(2, -1)$, and $M(-2, -1)$.

SOLUTION

Step 1 Draw the parallelogram in a coordinate plane by plotting the vertices and connecting them.



Step 2 Find the length of the base and the height.

Base

Let $\overline{JK}$ be the base. Use the Ruler Postulate to find the length of $\overline{JK}$.

$JK = |1 - (-3)| = |4| = 4$ Ruler Postulate

So, the length of the base is 4 units.

Height

Let the height be the distance from point M to $\overline{JK}$. By counting grid lines, you can determine that the height is 6 units.

Step 3 Substitute the values for the base and height into the formula for the area of a parallelogram.

$A = bh$ Write the formula for area of a parallelogram.

$= 4(6)$ Substitute.

$= 24$ Multiply.

ANOTHER WAY

You can also find the area of ▱*JKLM* by decomposing the parallelogram into a rectangle and two triangles, then finding the sum of their areas.

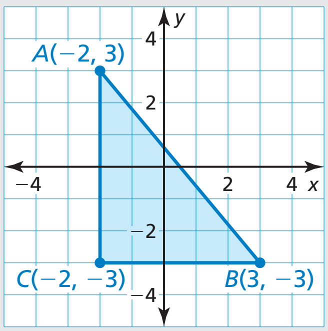
▶ So, the area of ▱$JKLM$ is 24 square units.

SELF-ASSESSMENT

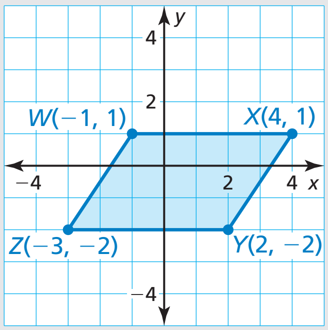
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Find the area of the polygon with the given vertices.

7.



8.



9. $N(-1, 1)$, $P(2, 1)$, $Q(2, -2)$, $R(-1, -2)$

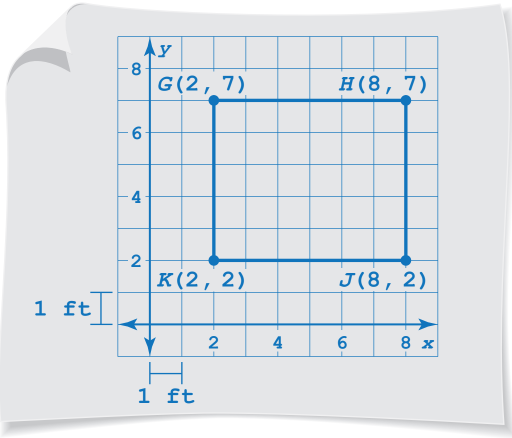
10. $K(-3, 3)$, $L(3, 3)$, $M(3, -1)$, $N(-3, -1)$

EXAMPLE 4 Modeling Real Life

WATCH

GO DIGITAL

You are building a shed in your backyard. The diagram shows the four vertices of the shed floor. Each unit in the coordinate plane represents 1 foot. Find the perimeter and the area of the floor of the shed.



SOLUTION

1. **Understand the Problem** You are given the coordinates of the vertices of the shed floor. You need to find the perimeter and the area of the floor.

2. **Make a Plan** The floor of the shed is rectangular, so use the coordinates of the vertices to find the length and the width. Then use formulas to find the perimeter and area.

3. **Solve and Check**

   Step 1 Find the length and the width.

   Length $GH = |8 - 2| = 6$ Ruler Postulate

   Width $KG = |7 - 2| = 5$ Ruler Postulate

   The shed has a length of 6 feet and a width of 5 feet.

   Step 2 Substitute the values for the length and width into the formulas for the perimeter P and area A of a rectangle.

$P = 2\ell + 2w$	Write formulas.	$A = \ell w$
$= 2(6) + 2(5)$	Substitute.	$= 6(5)$
$= 22$	Evaluate.	$= 30$

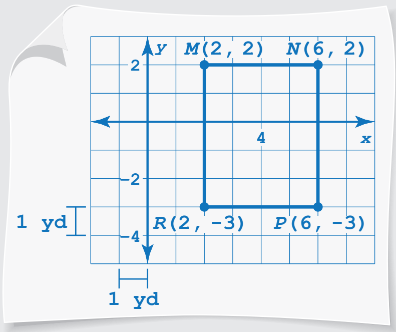
▶ The perimeter of the floor of the shed is 22 feet and the area is 30 square feet.

Check To check the perimeter, count the grid lines around the floor of the shed. There are 22 grid lines. To check the area, count the number of grid squares that make up the floor. There are 30 grid squares. ✓

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

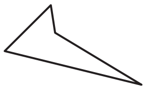
11. You are building a patio in your school's courtyard. The diagram shows the four vertices of the patio. Each unit in the coordinate plane represents 1 yard. Find the perimeter and the area of the patio.

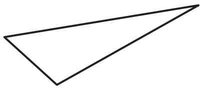


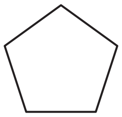
1.4 Practice WITH CalcChat® AND CalcView®

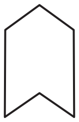
GO DIGITAL

In Exercises 1–4, classify the polygon by the number of sides. Tell whether it is *convex* or *concave*. *Example 1*

1. 

2. 

3. 

4. 

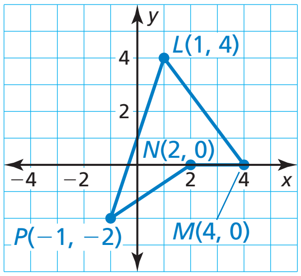
In Exercises 5–10, find the perimeter of the polygon with the given vertices. *Example 2*

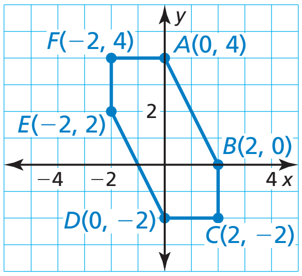
5. $G(2, 4), H(2, -3), J(-2, -3), K(-2, 4)$

6. $Q(-3, 2), R(1, 2), S(1, -2), T(-3, -2)$

7. $U(-2, 4), V(3, 4), W(3, -4)$

8. $X(-1, 3), Y(3, 0), Z(-1, -2)$

9. 

10. 

In Exercises 11–14, find the area of the polygon with the given vertices. *Example 3*

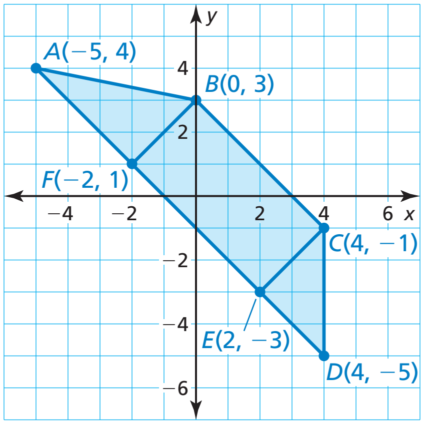
11. $E(3, 1), F(3, -2), G(-2, -2)$

12. $J(-3, 4), K(4, 4), L(3, -3)$

13. $W(0, 0), X(0, 3), Y(-3, 3), Z(-3, 0)$

14. $N(-4, 1), P(1, 1), Q(3, -1), R(-2, -1)$

In Exercises 15–18, use the diagram to find the perimeter and the area of the polygon.



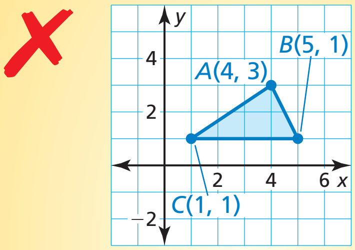
15. $\triangle CDE$

16. $\triangle ABF$

17. rectangle $BCEF$

18. quadrilateral $ABCD$

19. ERROR ANALYSIS Describe and correct the error in finding the area of the triangle.



$b = |5 - 1| = 4$

$h = \sqrt{(5-4)^2 + (1-3)^2} = \sqrt{5}$

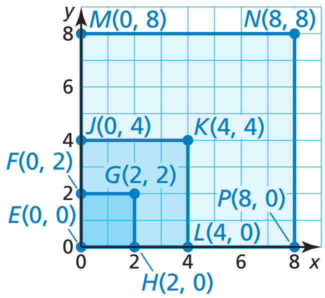
$A = \frac{1}{2}bh = \frac{1}{2}(4)(\sqrt{5}) = 2\sqrt{5}$

The area is $2\sqrt{5}$ square units.

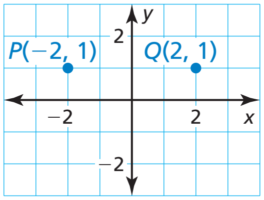
20. MP **REPEATED REASONING** Use the diagram.

a. Find the perimeter and area of each square.

b. What happens to the area of a square when its perimeter increases by a factor of n?



COLLEGE PREP **In Exercises 21 and 22, use the diagram.**



21. Determine which point is the remaining vertex of a triangle with an area of 4 square units.

Ⓐ $R(2, 0)$ Ⓑ $S(-2, -1)$

Ⓒ $T(-1, 0)$ Ⓓ $U(2, -2)$

22. Determine which points are the remaining vertices of a rectangle with a perimeter of 14 units.

Ⓐ $A(2, -1)$ and $B(-2, -1)$

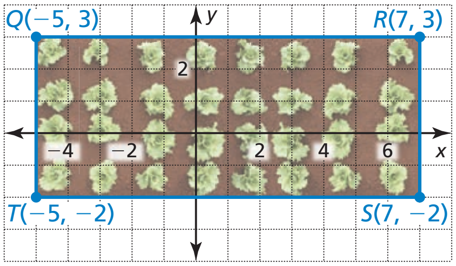
Ⓑ $C(-1, -2)$ and $D(1, -2)$

Ⓒ $E(-2, -2)$ and $F(2, -2)$

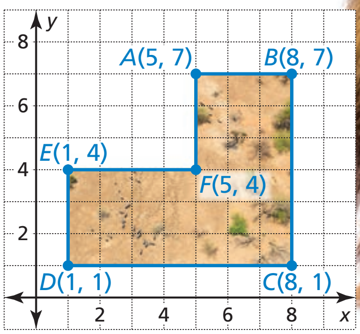
Ⓓ $G(2, 0)$ and $H(-2, 0)$

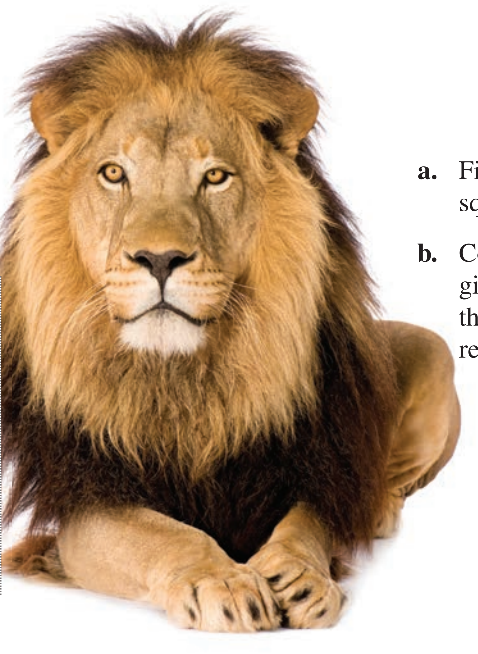
23. **MODELING REAL LIFE** You are building a school garden. The diagram shows the four vertices of the garden. Each unit in the coordinate plane represents 1 foot. Find the perimeter and the area of the garden.

Example 4



24. **MODELING REAL LIFE** The diagram shows the vertices of a lion sanctuary. Each unit in the coordinate plane represents 100 feet. Find the perimeter and the area of the sanctuary.





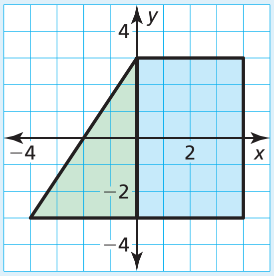
25. **MODELING REAL LIFE** You and your friend hike to a waterfall that is 4 miles east of where you left your bikes. You then hike to a lookout point that is 2 miles north of your bikes. From the lookout point, you return to your bikes.

a. About how far do you hike? Assume you travel along straight paths.

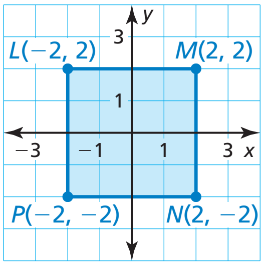
b. From the waterfall, your friend hikes to a wishing well before going to the lookout point and returning to the bikes. The wishing well is 3 miles north and 2 miles west of the lookout point. About how far does your friend hike?

GO DIGITAL

26. **HOW DO YOU SEE IT?**
Without performing any calculations, determine whether the triangle or the rectangle has a greater area. Which polygon has a greater perimeter? Explain your reasoning.



27. **ANALYZING RELATIONSHIPS** Use the diagram.



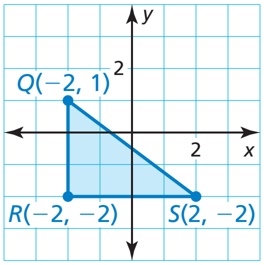
a. Find the perimeter and the area of the square.

b. Connect the midpoints of the sides of the given square to make a quadrilateral. Is this quadrilateral a square? Explain your reasoning.

c. Find the perimeter and the area of the quadrilateral you made in part (b). Compare this area to the area of the square you found in part (a).

28. **CONNECTING CONCEPTS** The lines $y_1 = 2x - 6$, $y_2 = -3x + 4$, and $y_3 = -\frac{1}{2}x + 4$ intersect to form the sides of a right triangle. Find the perimeter and the area of the triangle.

29. **MAKING AN ARGUMENT** Will a rectangle that has the same perimeter as $\triangle QRS$ have the same area as the triangle? Explain your reasoning.



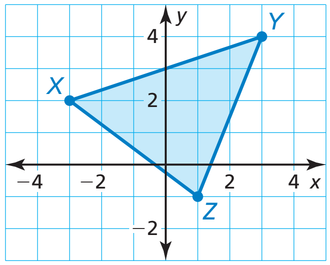
30. **THOUGHT PROVOKING**
A café that has an area of 350 square feet is being expanded to occupy an adjacent space that has an area of 150 square feet. Draw a diagram of the remodeled café in a coordinate plane.

31. **MP REASONING** Triangle ABC has a perimeter of 12 units. The vertices of the triangle are $A(x, 2)$, $B(2, -2)$, and $C(-1, 2)$. Find the value of x.

GO DIGITAL

32. **PERFORMANCE TASK** As a graphic designer, your job is to create a company logo that includes at least two different polygons and has an area of at least 50 square units. Draw your logo in a coordinate plane and record its perimeter and area. Describe the company and create a proposal explaining how your logo relates to the company.

33. **DIG DEEPER** Find the area of $\triangle XYZ$. (*Hint:* Draw a rectangle whose sides contain points X, Y, and Z.)



REVIEW & REFRESH

WATCH

34. Does the table represent a *linear* or *nonlinear* function? Explain.

x	−1	0	1	2	3
y	−9	−7	−5	−3	−1

In Exercises 35–38, solve the equation.

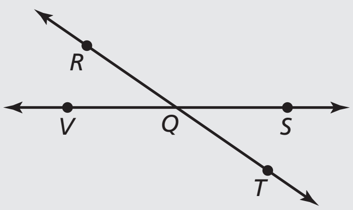
35. $3x - 7 = 2$

36. $4 = 9 + 5x$

37. $x + 4 = x - 12$

38. $\frac{x + 1}{2} = 4x - 3$

In Exercises 39 and 40, use the diagram.



39. Give another name for $\overline{RT}$.

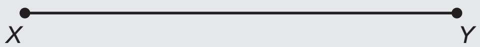
40. Name two pairs of opposite rays.

In Exercises 41 and 42, the endpoints of a segment are given. Find the coordinates of the midpoint *M* and the length of the segment.

41. $J(4, 3)$ and $K(2, -3)$

42. $L(-4, 5)$ and $N(5, -3)$

43. Use a compass and straightedge to construct a copy of the line segment.



44. **MODELING REAL LIFE** You deposit \$200 into a savings account that earns 5% annual interest compounded quarterly. Write a function that represents the balance y (in dollars) after t years.

In Exercises 45 and 46, graph the function. Then describe the transformations from the graph of $f(x) = |x|$ to the graph of the function.

45. $g(x) = |x - 4| + 5$

46. $h(x) = -3|x + 1|$

47. Find the perimeter and the area of $\square ABCD$ with vertices $A(3, 5)$, $B(6, 5)$, $C(4, -1)$, and $D(1, -1)$.

1.5 Measuring and Constructing Angles

Learning Target Measure, construct, and describe angles.

Success Criteria
- I can measure and classify angles.
- I can construct congruent angles.
- I can find angle measures.
- I can construct an angle bisector.

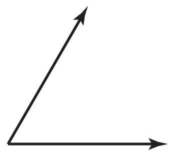
EXPLORE IT! Analyzing a Geometric Figure

Work with a partner.

Math Practice

Recognize Usefulness of Tools
Give some real-life situations that can be represented by these types of angles. Research different ways you can measure these types of angles in real life.

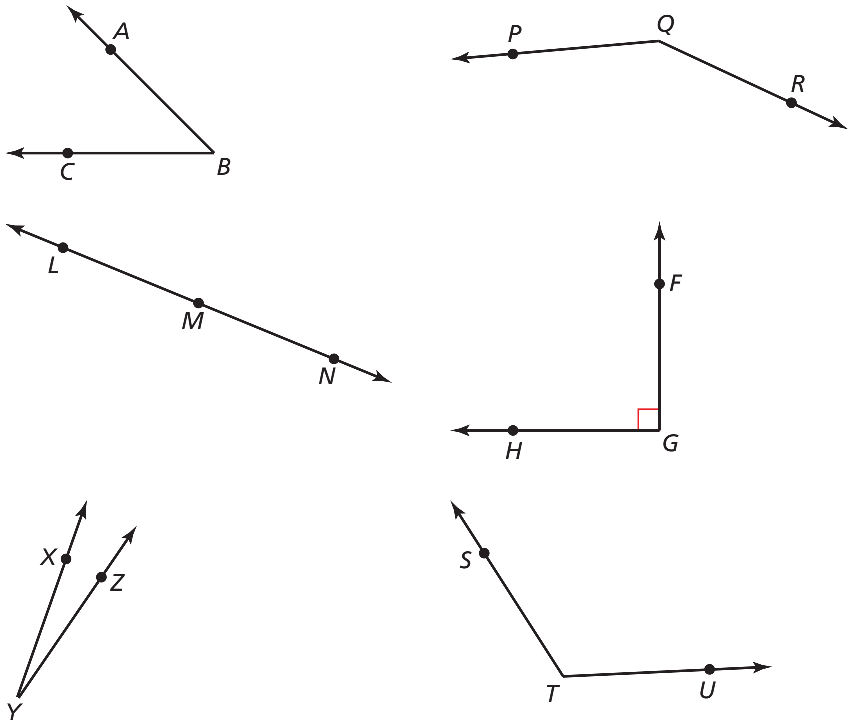
a. Identify the figure shown at the right. Then define it in your own words.



b. Label and name the figure. Then compare your results with those of your classmates.

c. How can you *measure* the figure?

d. Describe each angle below. How would you group these angles? Explain.



e. MP **CHOOSE TOOLS** Construct a copy of an angle from part (d). Explain your method.

f. Construct each of the following. Explain your method.

i. An angle that is twice the measure of the angle in part (a)

ii. Separate the angle in part (a) into two angles with the same measure.

Vocabulary

angle, *p. 36*
vertex, *p. 36*
sides of an angle, *p. 36*
interior of an angle, *p. 36*
exterior of an angle, *p. 36*
measure of an angle, *p. 36*
acute angle, *p. 37*
right angle, *p. 37*
obtuse angle, *p. 37*
straight angle, *p. 37*
congruent angles, *p. 38*
angle bisector, *p. 40*

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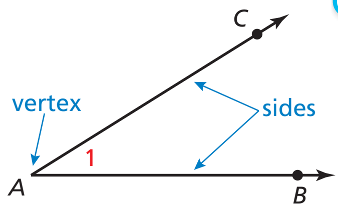
Naming Angles

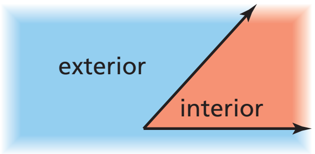
An **angle** is a set of points consisting of two different rays that have the same endpoint, called the **vertex**. The rays are the **sides** of the angle.



You can name an angle in several different ways. The symbol $\angle$ represents an angle.

- Use its vertex, such as $\angle A$.
- Use a point on each ray and the vertex, such as $\angle BAC$ or $\angle CAB$.
  Make sure the vertex is the middle letter.
- Use a number, such as $\angle 1$.

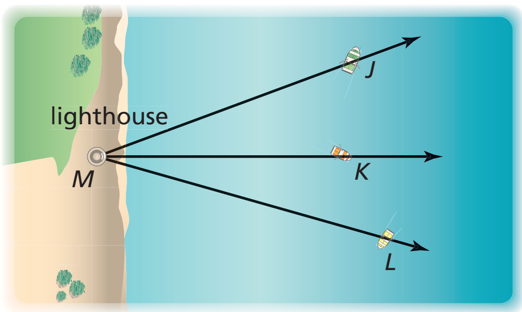
The region that contains all the points between the sides of the angle is the **interior of the angle**. The region that contains all the points outside the angle is the **exterior of the angle**.



EXAMPLE 1 Naming Angles

WATCH INFO

A lighthouse keeper measures the angles formed by the lighthouse at point *M* and three boats. Name three angles shown in the diagram.



SOLUTION

$\angle JMK$ or $\angle KMJ$

$\angle KML$ or $\angle LMK$

$\angle JML$ or $\angle LMJ$

COMMON ERROR

When a point is the vertex of more than one angle, you cannot use the vertex alone to name the angle.

Measuring and Classifying Angles

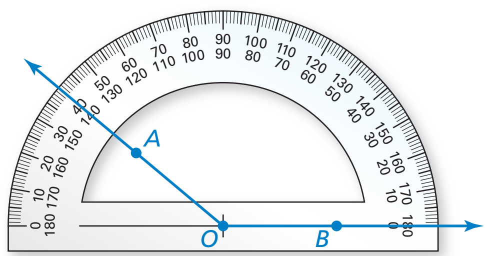
A protractor helps you approximate the *measure* of an angle. The measure is usually given in *degrees*.

POSTULATE

1.3 Protractor Postulate

Consider $\overleftrightarrow{OB}$ and a point A on one side of $\overleftrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180.

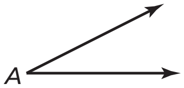
The **measure** of $\angle AOB$, which can be written as $m\angle AOB$, is equal to the absolute value of the difference between the real numbers matched with $\overrightarrow{OA}$ and $\overrightarrow{OB}$ on a protractor.



You can classify angles according to their measures.

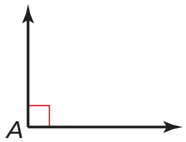
KEY IDEA

Types of Angles



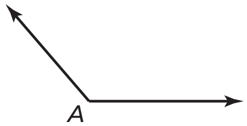
acute angle

Measures greater than 0° and less than 90°



right angle

Measures 90°



obtuse angle

Measures greater than 90° and less than 180°

straight angle

Measures 180°

EXAMPLE 2 Measuring and Classifying Angles

COMMON ERROR

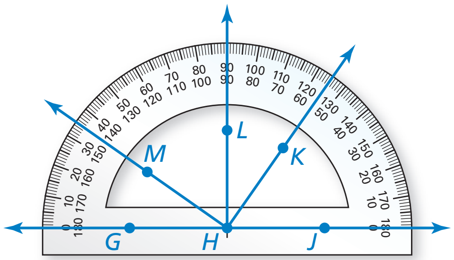
Most protractors have an inner and an outer scale. When measuring, make sure you are using the correct scale.

Find the measure of each angle. Then classify the angle.

a. $\angle GHK$

b. $\angle JHL$

c. $\angle LHK$



SOLUTION

a. $\overrightarrow{HG}$ lines up with 0° on the outer scale of the protractor. $\overrightarrow{HK}$ passes through 125° on the outer scale. So, $m\angle GHK = 125°$. It is an *obtuse* angle.

b. $\overrightarrow{HJ}$ lines up with 0° on the inner scale of the protractor. $\overrightarrow{HL}$ passes through 90°. So, $m\angle JHL = 90°$. It is a *right* angle.

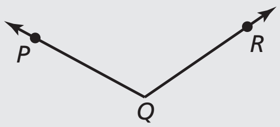
c. $\overrightarrow{HL}$ passes through 90°. $\overrightarrow{HK}$ passes through 55° on the inner scale. So, $m\angle LHK = |90 - 55| = 35°$. It is an *acute* angle.

SELF-ASSESSMENT

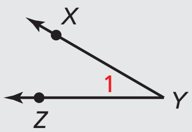
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

Write three names for the angle.

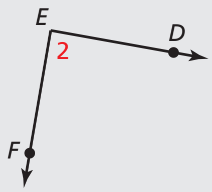
1.



2.

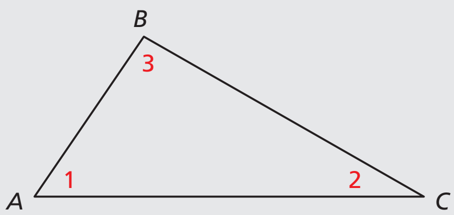


3.



4. WHICH ONE DOESN'T BELONG? Which angle name does *not* belong with the other three? Explain your reasoning.

$\angle BCA$ $\angle BAC$ $\angle 1$ $\angle CAB$



Use the diagram in Example 2 to find the measure of the angle. Then classify the angle.

5. $\angle JHM$

6. $\angle MHK$

7. $\angle MHL$

Identifying Congruent Angles

GO DIGITAL

You can use a compass and straightedge to construct an angle that has the same measure as a given angle.

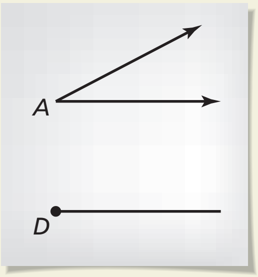
CONSTRUCTION Copying an Angle

WATCH

Use a compass and straightedge to construct an angle that has the same measure as $\angle A$. In this construction, the *center* of an arc is the point where the compass point rests. The *radius* of an arc is the distance from the center of the arc to a point on the arc drawn by the compass.

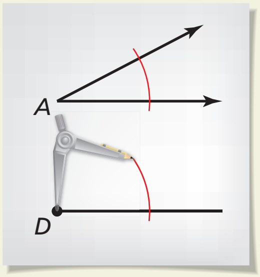
SOLUTION

Step 1



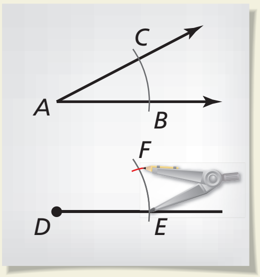
Draw a segment Draw an angle such as $\angle A$, as shown. Then draw a segment. Label point D on the segment.

Step 2



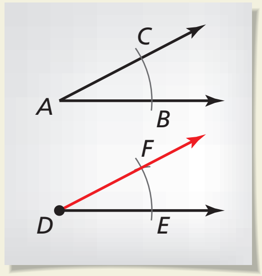
Draw arcs Draw an arc with center A. Using the same radius, draw an arc with center D.

Step 3



Draw an arc Label B, C, and E. Draw an arc with radius BC and center E. Label the intersection F.

Step 4



Draw a ray Draw $\overrightarrow{DF}$. $\angle D$ has the same measure as $\angle A$.

Two angles are **congruent angles** when they have the same measure. In the construction above, $\angle A$ and $\angle D$ are congruent angles. So,

$m\angle A = m\angle D$ The measure of angle A is *equal to* the measure of angle D.

and

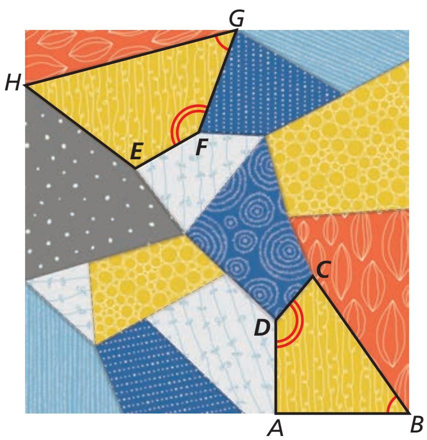
$\angle A \cong \angle D.$ Angle A is *congruent to* angle D.

EXAMPLE 3 Identifying Congruent Angles

WATCH

a. Identify the congruent angles labeled in the quilt design.

b. $m\angle ADC = 140°$. What is $m\angle EFG$?



SOLUTION

a. There are two pairs of congruent angles:

$\angle ABC \cong \angle FGH$

and

$\angle ADC \cong \angle EFG.$

b. Because $\angle ADC \cong \angle EFG$, $m\angle ADC = m\angle EFG$.

▶ So, $m\angle EFG = 140°$.

READING

In a diagram, matching arcs indicate congruent angles. When there is more than one pair of congruent angles, use multiple arcs.

GO DIGITAL

Using the Angle Addition Postulate

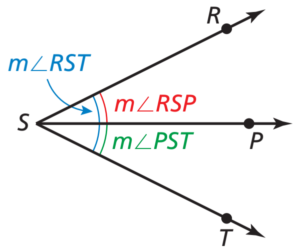
POSTULATE

1.4 Angle Addition Postulate

Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle RSP$ and $\angle PST$.

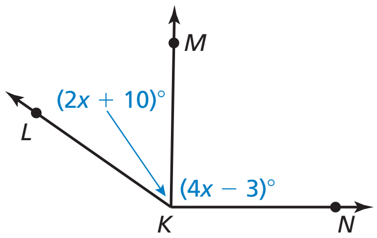
Symbols If P is in the interior of $\angle RST$, then

$$m\angle RST = m\angle RSP + m\angle PST.$$



EXAMPLE 4 Finding Angle Measures

WATCH



Given that $m\angle LKN = 145°$, find $m\angle LKM$ and $m\angle MKN$.

SOLUTION

Step 1 Write and solve an equation to find the value of x.

$m\angle LKN = m\angle LKM + m\angle MKN$	Angle Addition Postulate
$145° = (2x + 10)° + (4x - 3)°$	Substitute angle measures.
$145 = 6x + 7$	Combine like terms.
$138 = 6x$	Subtract 7 from each side.
$23 = x$	Divide each side by 6.

Step 2 Evaluate the given expressions when $x = 23$.

$$m\angle LKM = (2x + 10)° = (2 \cdot 23 + 10)° = 56°$$

$$m\angle MKN = (4x - 3)° = (4 \cdot 23 - 3)° = 89°$$

▶ So, $m\angle LKM = 56°$ and $m\angle MKN = 89°$.

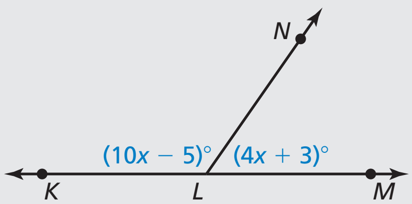
SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

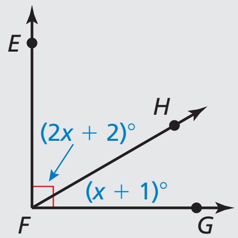
8. Without measuring, determine whether $\angle DAB$ and $\angle FEH$ in Example 3 appear to be congruent. Explain your reasoning. Use a protractor to verify your answer.

Find the indicated angle measures.

9. Given that $\angle KLM$ is a straight angle, find $m\angle KLN$ and $m\angle NLM$.



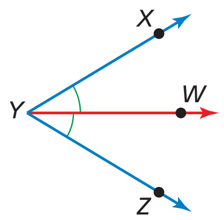
10. Given that $\angle EFG$ is a right angle, find $m\angle EFH$ and $m\angle HFG$.



Bisecting Angles

An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the figure, $\overrightarrow{YW}$ bisects $\angle XYZ$, so $\angle XYW \cong \angle ZYW$.

You can use a compass and straightedge to bisect an angle.



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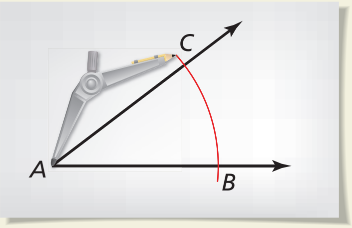
CONSTRUCTION **Bisecting an Angle**

Construct an angle bisector of $\angle A$ with a compass and straightedge.

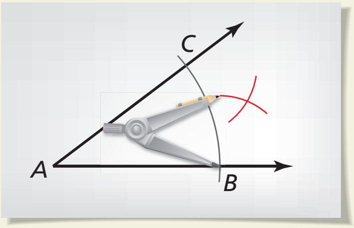
SOLUTION

Step 1



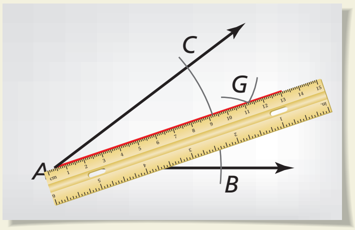
Draw an arc Draw an angle such as $\angle A$, as shown. Place the compass at A. Draw an arc with center A that intersects both sides of the angle. Label the intersections B and C.

Step 2



Draw arcs Draw an arc with center C. Using the same radius, draw an arc with center B.

Step 3



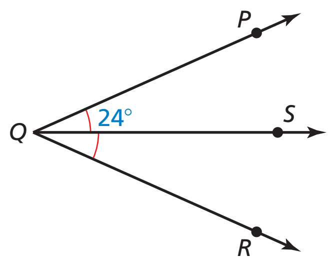
Draw a ray Label the intersection G. Use a straightedge to draw $\overrightarrow{AG}$. $\overrightarrow{AG}$ bisects $\angle A$.

EXAMPLE 5 **Using a Bisector to Find Angle Measures**

$\overrightarrow{QS}$ bisects $\angle PQR$, and $m\angle PQS = 24^\circ$. Find $m\angle PQR$.

SOLUTION

Step 1 Draw a diagram.



Step 2 Because $\overrightarrow{QS}$ bisects $\angle PQR$, $m\angle PQS = m\angle RQS$. So, $m\angle RQS = 24^\circ$. Use the Angle Addition Postulate to find $m\angle PQR$.

$m\angle PQR = m\angle PQS + m\angle RQS$ — Angle Addition Postulate

$= 24^\circ + 24^\circ$ — Substitute angle measures.

$= 48^\circ$ — Add.

▶ So, $m\angle PQR = 48^\circ$.

SELF-ASSESSMENT 1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

11. Angle MNP is a straight angle, and $\overrightarrow{NQ}$ bisects $\angle MNP$. Draw $\angle MNP$ and $\overrightarrow{NQ}$. Use matching arcs to indicate congruent angles in your diagram. Find the angle measures of these congruent angles.

1.5 Practice WITH CalcChat® AND CalcView®

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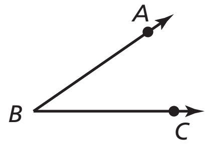
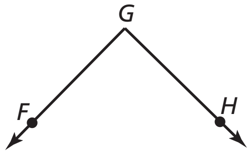
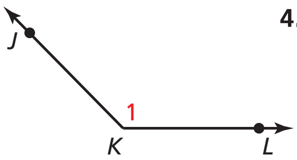
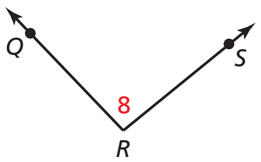
In Exercises 1–4, write three names for the angle.

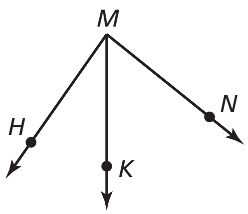
1. 

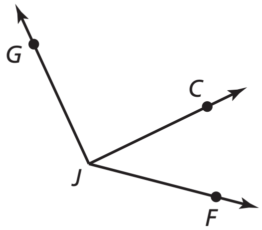
2. 

3. 

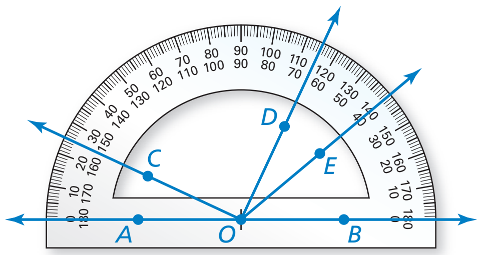
4. 

In Exercises 5 and 6, name three different angles in the diagram. *Example 1*

5. 

6. 

In Exercises 7–10, find the angle measure. Then classify the angle. *Example 2*



7. $m\angle BOD$

8. $m\angle AOE$

9. $m\angle COE$

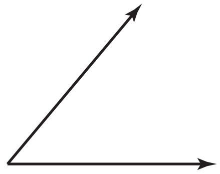
10. $m\angle COD$

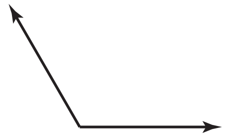
ERROR ANALYSIS **In Exercises 11 and 12, describe and correct the error in finding the angle measure. Use the diagram from Exercises 7–10.**

11. ✗ $m\angle BOC = 25°$

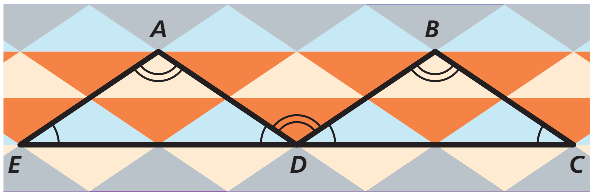
12. ✗ $m\angle DOE = 65°$

CONSTRUCTION **In Exercises 13 and 14, use a compass and straightedge to copy the angle.**

13. 

14. 

In Exercises 15–18, $m\angle AED = 34°$ and $m\angle EAD = 112°$. *Example 3*



15. Identify the angles congruent to $\angle AED$.

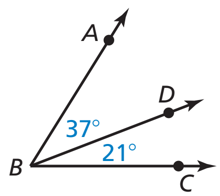
16. Identify the angles congruent to $\angle EAD$.

17. Find $m\angle BDC$.

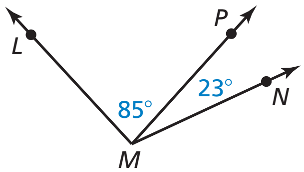
18. Find $m\angle ADB$.

In Exercises 19–22, find the indicated angle measure.

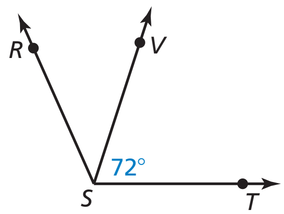
19. Find $m\angle ABC$.



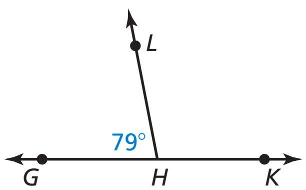
20. Find $m\angle LMN$.



21. $m\angle RST = 114°$. Find $m\angle RSV$.

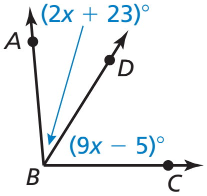


22. $\angle GHK$ is a straight angle. Find $m\angle LHK$.

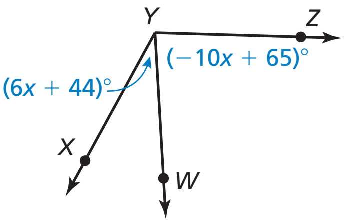


In Exercises 23–28, find the indicated angle measures. *Example 4*

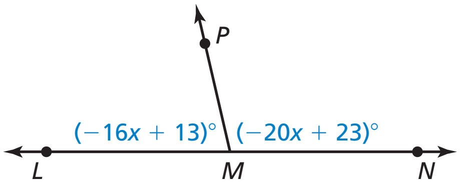
23. $m\angle ABC = 95^\circ$. Find $m\angle ABD$ and $m\angle DBC$.



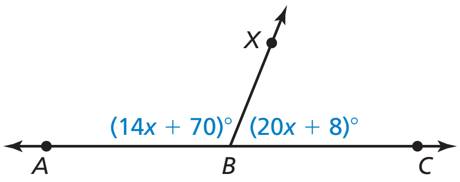
24. $m\angle XYZ = 117^\circ$. Find $m\angle XYW$ and $m\angle WYZ$.



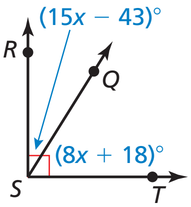
25. $\angle LMN$ is a straight angle. Find $m\angle LMP$ and $m\angle NMP$.



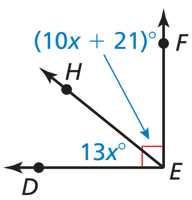
26. $\angle ABC$ is a straight angle. Find $m\angle ABX$ and $m\angle CBX$.



27. Find $m\angle RSQ$ and $m\angle TSQ$.



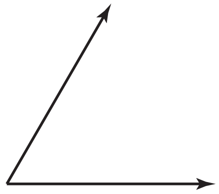
28. Find $m\angle DEH$ and $m\angle FEH$.



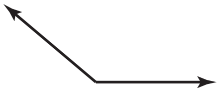
GO DIGITAL

CONSTRUCTION **In Exercises 29 and 30, copy the angle. Then construct the angle bisector with a compass and straightedge.**

29.



30.



In Exercises 31–34, $\overrightarrow{FH}$ bisects $\angle EFG$. Find the indicated angle measures. *Example 5*

31. $m\angle EFH = 63^\circ$. Find $m\angle GFH$ and $m\angle EFG$.

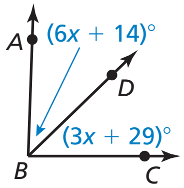
32. $m\angle GFH = 71^\circ$. Find $m\angle EFH$ and $m\angle EFG$.

33. $m\angle EFG = 124^\circ$. Find $m\angle EFH$ and $m\angle GFH$.

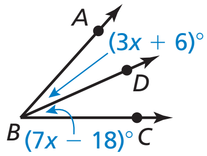
34. $m\angle EFG = 119^\circ$. Find $m\angle EFH$ and $m\angle GFH$.

In Exercises 35–38, $\overrightarrow{BD}$ bisects $\angle ABC$. Find $m\angle ABD$, $m\angle CBD$, and $m\angle ABC$.

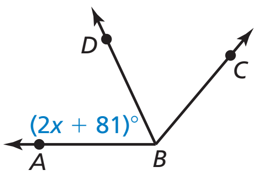
35.



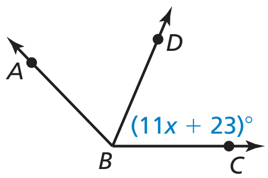
36.



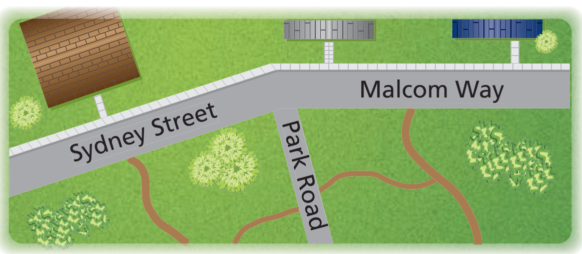
37. $m\angle ABC = (2 - 16x)^\circ$



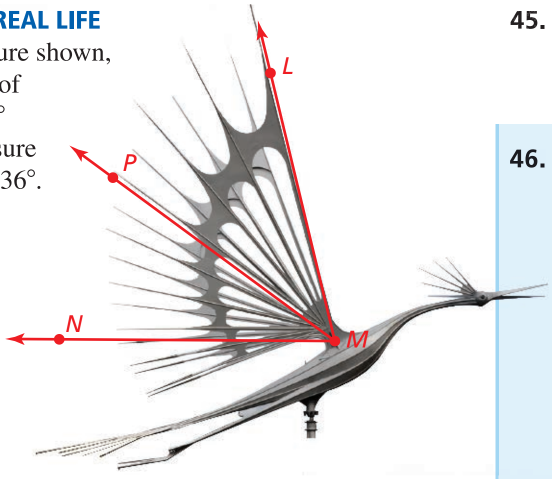
38. $m\angle ABC = (25x + 34)^\circ$



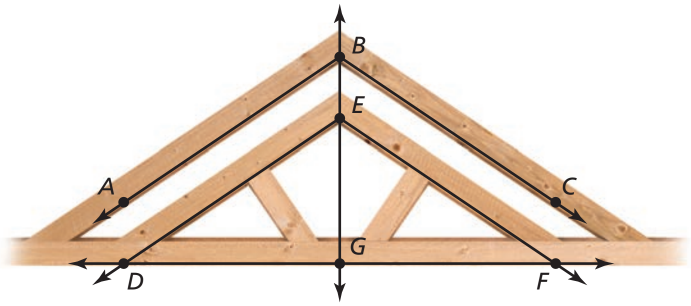
39. **MODELING REAL LIFE** The map shows the intersections of three roads. Malcom Way intersects Sydney Street at an angle of 162°. Park Road intersects Sydney Street at an angle of 87°. Find the angle at which Malcom Way intersects Park Road.



40. MODELING REAL LIFE In the sculpture shown, the measure of $\angle LMN$ is 76° and the measure of $\angle PMN$ is 36°. What is the measure of $\angle LMP$?



MODELING REAL LIFE **In Exercises 41 and 42, use the diagram of the roof truss.**



41. $\overrightarrow{BG}$ bisects $\angle ABC$ and $\angle DEF$, $m\angle ABC = 112°$, and $\angle ABC \cong \angle DEF$. Find the measure of each angle.

a. $\angle DEF$

b. $\angle ABG$

c. $\angle CBG$

d. $\angle DEG$

42. $\angle DGF$ is a straight angle, and $\overrightarrow{GB}$ bisects $\angle DGF$. Find $m\angle DGE$ and $m\angle FGE$.

43. MP **NUMBER SENSE** Given $\angle ABC$, X is in the interior of the angle, $m\angle ABX$ is 12° more than 4 times $m\angle CBX$, and $m\angle ABC = 92°$. Find $m\angle ABX$ and $m\angle CBX$.

44. CRITICAL THINKING In a coordinate plane, the ray from the origin through (4, 0) forms one side of an angle. Use the numbers below as x- and y-coordinates to create each type of angle.

−2	−1	0	1	2

a. acute angle

b. right angle

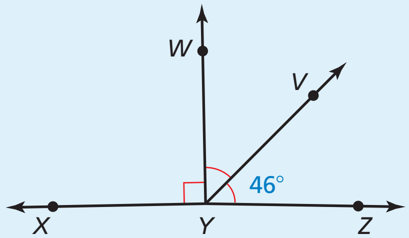
c. obtuse angle

d. straight angle

45. MAKING AN ARGUMENT Is it possible for a straight angle to consist of two obtuse angles? Explain your reasoning.

GO DIGITAL

46. HOW DO YOU SEE IT?
Is it possible for $\angle XYZ$ to be a straight angle? Explain your reasoning. If it is not possible, what can you change in the diagram so that $\angle XYZ$ is a straight angle?



47. ABSTRACT REASONING Classify the angles that result from bisecting each type of angle.

a. acute angle

b. right angle

c. obtuse angle

d. straight angle

48. ABSTRACT REASONING Classify the angles that result from drawing a ray from the vertex through a point in the interior of each type of angle. Include all possibilities, and explain your reasoning.

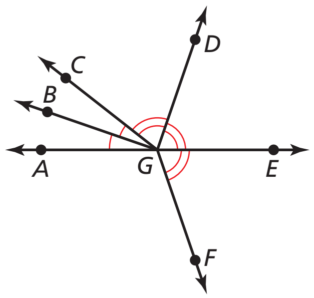
a. acute angle

b. right angle

c. obtuse angle

d. straight angle

49. COLLEGE PREP In the diagram, $m\angle AGC = 38°$, $m\angle CGD = 71°$, and $m\angle FGC = 147°$. Which of the following statements are true? Select all that apply.



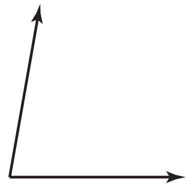
Ⓐ $m\angle AGB = 19°$

Ⓑ $m\angle DGF = 142°$

Ⓒ $m\angle AGF = 128°$

Ⓓ $\angle BGD$ is a right angle.

50. **MP REASONING** Copy the angle. Then construct an angle with a measure that is $\frac{1}{4}$ the measure of the given angle. Explain your reasoning.



51. **ANALYZING RELATIONSHIPS** $\overrightarrow{SQ}$ bisects $\angle RST$, $\overrightarrow{SP}$ bisects $\angle RSQ$, and $\overrightarrow{SV}$ bisects $\angle RSP$. The measure of $\angle VSP$ is 17°. Find $m\angle TSQ$. Explain.

GO DIGITAL

52. **THOUGHT PROVOKING**
How many times between 12 A.M. and 12 P.M. do the minute hand and hour hand of a clock form a right angle? (Be sure to consider how the hour hand moves, in addition to how the minute hand moves.)

REVIEW & REFRESH

WATCH

53. Find the perimeter and the area of $\triangle ABC$ with vertices $A(-1, 1)$, $B(2, 1)$, and $C(1, -2)$.

In Exercises 54–56, solve the equation.

54. $3x + 15 + 4x - 9 = 90$

55. $\frac{1}{2}(4x + 6) - 11 = 5x + 7$

56. $3(6 - 8x) = 2(-12x + 9)$

In Exercises 57–60, simplify the expression.

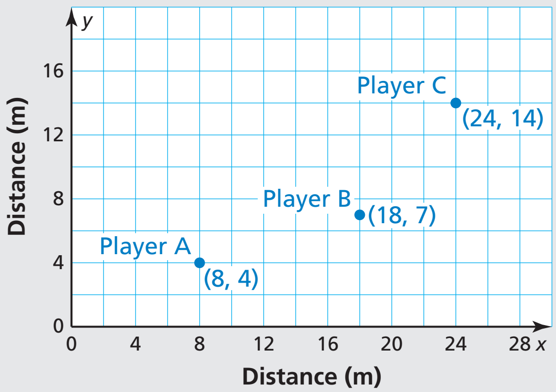
57. $\sqrt{160}$

58. $\sqrt[3]{135}$

59. $\sqrt{\frac{21}{100}}$

60. $\frac{\sqrt{11}}{\sqrt{5}}$

61. **MODELING REAL LIFE** The positions of three players during part of a water polo match are shown. Player A throws the ball to Player B, who then throws the ball to Player C.



a. Who throws the ball farther, Player A or B?

b. About how far would Player A have to throw the ball to throw it directly to Player C?

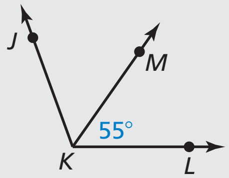
In Exercises 62 and 63, graph the inequality in a coordinate plane.

62. $x \geq -2.5$

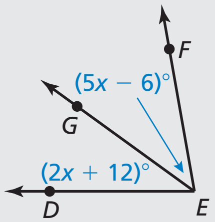
63. $y < -\frac{1}{3}x + 2$

In Exercises 64 and 65, find the indicated angle measures.

64. $\overrightarrow{KM}$ bisects $\angle JKL$. Find $m\angle JKM$ and $m\angle JKL$.



65. $m\angle DEF = 76°$. Find $m\angle DEG$ and $m\angle GEF$.



In Exercises 66 and 67, solve the system using any method. Explain your choice of method.

66. $2x + 3y = 3$
$x = y - 11$

67. $3x - 4y = 24$
$-5x + 2y = -26$

68. Graph $y = \begin{cases} -x, & \text{if } x \leq -1 \\ 2x - 3, & \text{if } x > -1 \end{cases}$. Find the domain and range.

69. Point Y is between points X and Z on $\overline{XZ}$. $XY = 27$ and $YZ = 8$. Find XZ.

1.6 Describing Pairs of Angles

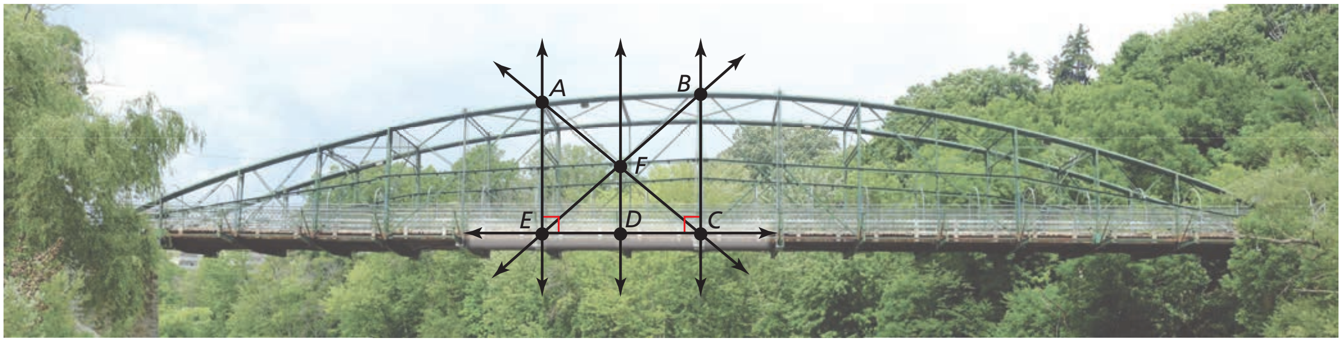
Learning Target Identify and use pairs of angles.

Success Criteria

- I can identify complementary and supplementary angles.
- I can identify linear pairs and vertical angles.
- I can find angle measures in pairs of angles.

EXPLORE IT! Identifying Pairs of Angles

Work with a partner. The Blackfriars Street Bridge in London, Ontario, Canada, is a bowstring arch-truss bridge. Use the diagram to complete parts (a)–(c).



A bowstring arch-truss bridge is one of the rarest types of bridges. The bridge above was built in 1875. There are few bridges of this type remaining today.

Math Practice

Explain the Meaning
What does it mean to be *nonadjacent*? Identify a pair of nonadjacent angles in the diagram.

a. Identify a pair of the indicated angles. Do not use the same pair of angles twice.

i. complementary angles

ii. supplementary angles

iii. adjacent angles

iv. vertical angles

b. Suppose $\angle EDF$ and $\angle CDF$ are congruent. What can you conclude about $\overleftrightarrow{DF}$ and $\overleftrightarrow{EC}$? Explain.

c. What does it mean for two angles to form a *linear pair*? Identify a linear pair.

d. Research different bridge designs. Make sketches of the designs and identify pairs of complementary, supplementary, adjacent, and vertical angles. Why are these types of angles used when building bridges?

Using Complementary and Supplementary Angles

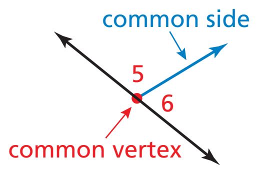
Vocabulary
adjacent angles, *p. 46*
complementary angles, *p. 46*
supplementary angles, *p. 46*
linear pair, *p. 48*
vertical angles, *p. 48*

Pairs of angles can have special relationships. The measurements of the angles or the positions of the angles in the pair determine the relationship.

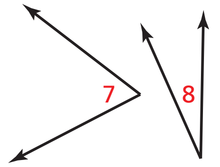
KEY IDEAS

Adjacent Angles

Adjacent angles are two angles that share a common vertex and side, but have no common interior points.

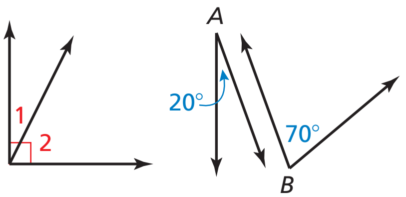


∠5 and ∠6 are adjacent angles.



∠7 and ∠8 are *nonadjacent* angles.

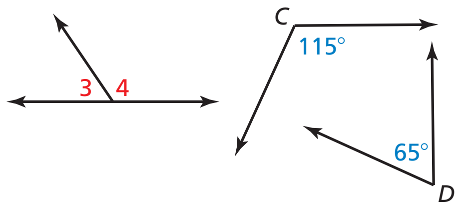
Complementary and Supplementary Angles



∠1 and ∠2   ∠*A* and ∠*B*

complementary angles

Complementary angles are two positive angles whose measures have a sum of 90°. Each angle is the *complement* of the other.



∠3 and ∠4   ∠*C* and ∠*D*

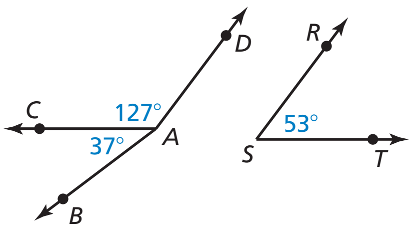
supplementary angles

Supplementary angles are two positive angles whose measures have a sum of 180°. Each angle is the *supplement* of the other.

STUDY TIP
Complementary angles and supplementary angles can be adjacent or nonadjacent.

EXAMPLE 1 Identifying Pairs of Angles

In the diagram, name a pair of adjacent angles, a pair of complementary angles, and a pair of supplementary angles.



COMMON ERROR
In Example 1, ∠*DAC* and ∠*DAB* share a common vertex and a common side, but they also share common interior points. So, they are *not* adjacent angles.

SOLUTION

∠*BAC* and ∠*CAD* share a common vertex and side, but have no common interior points. So, they are adjacent angles.

Because 37° + 53° = 90°, ∠*BAC* and ∠*RST* are complementary angles.

Because 127° + 53° = 180°, ∠*CAD* and ∠*RST* are supplementary angles.

GO DIGITAL

EXAMPLE 2 Finding Angle Measures

WATCH

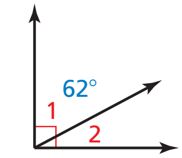
a. $\angle 1$ is a complement of $\angle 2$, and $m\angle 1 = 62^\circ$. Find $m\angle 2$.

b. $\angle 3$ is a supplement of $\angle 4$, and $m\angle 4 = 47^\circ$. Find $m\angle 3$.

SOLUTION

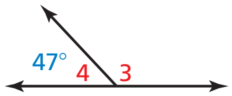
a. Draw a diagram with complementary adjacent angles to illustrate the relationship.

$$m\angle 2 = 90^\circ - m\angle 1 = 90^\circ - 62^\circ = 28^\circ$$



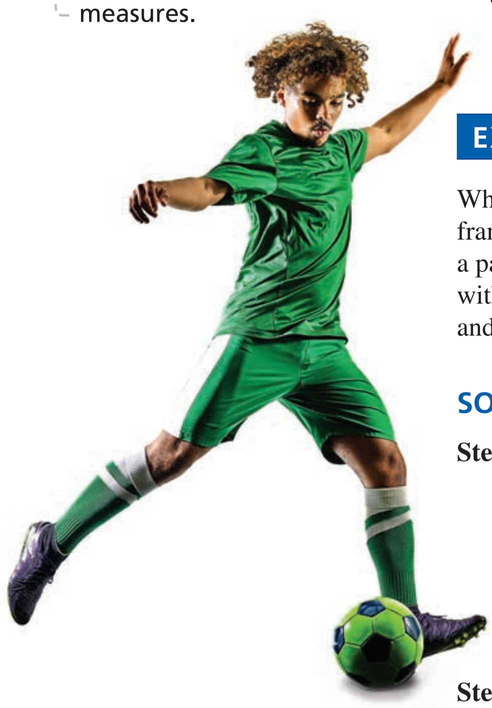
b. Draw a diagram with supplementary adjacent angles to illustrate the relationship.

$$m\angle 3 = 180^\circ - m\angle 4 = 180^\circ - 47^\circ = 133^\circ$$



COMMON ERROR

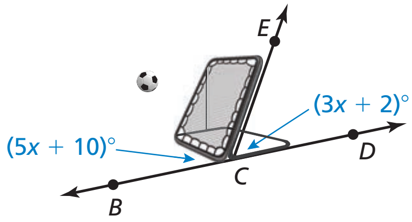
Do not confuse angle names with angle measures.



EXAMPLE 3 Modeling Real Life

WATCH

When viewed from the side, the frame of a ball-return net forms a pair of supplementary angles with the ground. Find $m\angle BCE$ and $m\angle ECD$.



SOLUTION

Step 1 Use the fact that the sum of the measures of supplementary angles is 180°.

$m\angle BCE + m\angle ECD = 180^\circ$	Write an equation.
$(5x + 10)^\circ + (3x + 2)^\circ = 180^\circ$	Substitute angle measures.
$8x + 12 = 180$	Combine like terms.
$x = 21$	Solve for x.

Step 2 Evaluate the given expressions when $x = 21$.

$$m\angle BCE = (5x + 10)^\circ = (5 \cdot 21 + 10)^\circ = 115^\circ$$

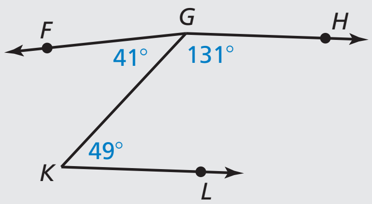
$$m\angle ECD = (3x + 2)^\circ = (3 \cdot 21 + 2)^\circ = 65^\circ$$

▶ So, $m\angle BCE = 115^\circ$ and $m\angle ECD = 65^\circ$.

SELF-ASSESSMENT

1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

In Exercises 1 and 2, use the diagram.



1. Name a pair of adjacent angles, a pair of complementary angles, and a pair of supplementary angles.
2. Are $\angle KGH$ and $\angle LKG$ adjacent angles? Are $\angle FGK$ and $\angle FGH$ adjacent angles? Explain.
3. $\angle 1$ is a complement of $\angle 2$, and $m\angle 2 = 5^\circ$. Find $m\angle 1$.
4. $\angle 3$ is a supplement of $\angle 4$, and $m\angle 3 = 148^\circ$. Find $m\angle 4$.
5. $\angle LMN$ and $\angle PQR$ are complementary angles. Find the measures of the angles when $m\angle LMN = (4x - 2)^\circ$ and $m\angle PQR = (9x + 1)^\circ$.

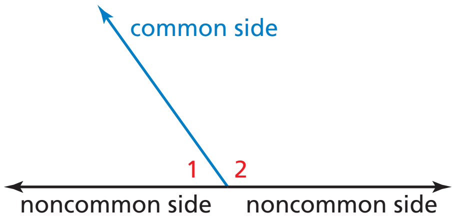
Using Other Angle Pairs

GO DIGITAL

KEY IDEAS

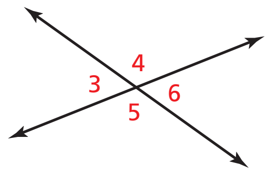
Linear Pairs and Vertical Angles

Two adjacent angles are a **linear pair** when their noncommon sides are opposite rays. The angles in a linear pair are supplementary angles.



$\angle 1$ and $\angle 2$ are a linear pair.

Two angles are **vertical angles** when their sides form two pairs of opposite rays.

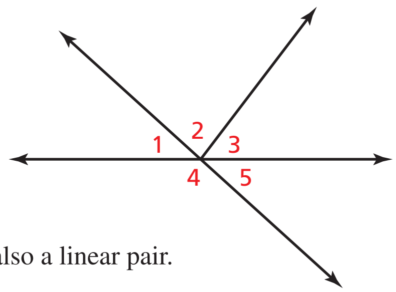


$\angle 3$ and $\angle 6$ are vertical angles.
$\angle 4$ and $\angle 5$ are vertical angles.

EXAMPLE 4 Identifying Angle Pairs

WATCH

Identify all the linear pairs and all the vertical angles in the diagram.



SOLUTION

To find linear pairs, look for adjacent angles whose noncommon sides are opposite rays.

▶ $\angle 1$ and $\angle 4$ are a linear pair. $\angle 4$ and $\angle 5$ are also a linear pair.

To find vertical angles, look for pairs of opposite rays.

▶ $\angle 1$ and $\angle 5$ are vertical angles.

> **COMMON ERROR**
> In Example 4, one side of $\angle 1$ and one side of $\angle 3$ are opposite rays. However, the angles are not a linear pair because they are *nonadjacent*.

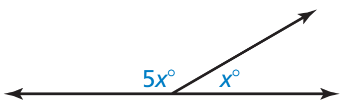
EXAMPLE 5 Finding Angle Measures in a Linear Pair

WATCH

Two angles form a linear pair. The measure of one angle is five times the measure of the other angle. Find the measure of each angle.

SOLUTION

Step 1 Draw a diagram. Let $x°$ be the measure of one angle. The measure of the other angle is $5x°$.



Step 2 Use the fact that the angles of a linear pair are supplementary to write an equation.

$x° + 5x° = 180°$ Write an equation.

$6x = 180$ Combine like terms.

$x = 30$ Divide each side by 6.

▶ The measures of the angles are $30°$ and $5(30°) = 150°$.

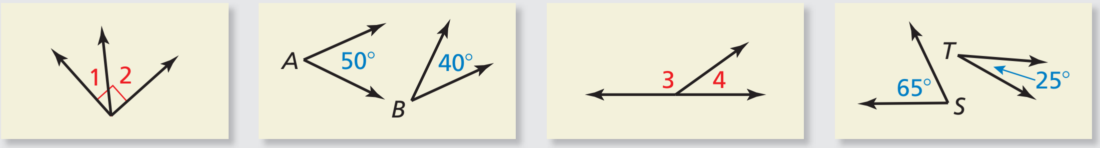
GO DIGITAL

SELF-ASSESSMENT

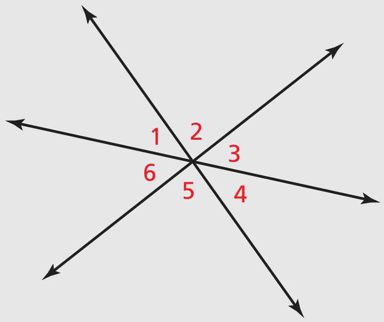
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

6. **WRITING** Explain the difference between adjacent angles and vertical angles.

7. **WHICH ONE DOESN'T BELONG?** Which one does *not* belong with the other three? Explain your reasoning.



8. Do any of the numbered angles in the diagram form a linear pair? Which angles are vertical angles? Explain your reasoning.



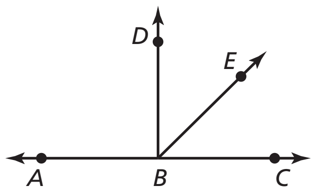
9. The measure of an angle is twice the measure of its complement. Find the measure of each angle.

10. Two angles form a linear pair. The measure of one angle is $1\frac{1}{2}$ times the measure of the other angle. Find the measure of each angle.

CONCEPT SUMMARY

Interpreting a Diagram

There are some things you can conclude from a diagram, and some you cannot. For example, here are some things you ***can* conclude** from the diagram.



YOU CAN CONCLUDE

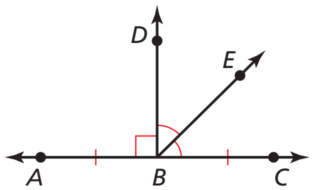
- All points shown are coplanar.
- Points *A*, *B*, and *C* are collinear, and *B* is between *A* and *C*.
- $\overleftrightarrow{AC}$, $\overrightarrow{BD}$, and $\overrightarrow{BE}$ intersect at point *B*.
- $\angle DBE$ and $\angle EBC$ are adjacent angles, and $\angle ABC$ is a straight angle.
- Point *E* lies in the interior of $\angle DBC$.

Here are some things you ***cannot* conclude** from the diagram above.

YOU CANNOT CONCLUDE

- $\overline{AB} \cong \overline{BC}$
- $\angle DBE \cong \angle EBC$
- $\angle ABD$ is a right angle.

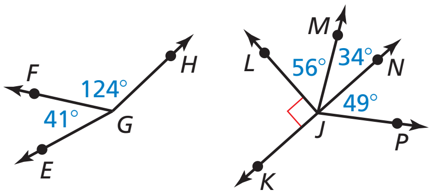
To make such conclusions, the information in the diagram at the right must be given.



1.6 Practice WITH CalcChat® AND CalcView®

GO DIGITAL

In Exercises 1–4, use the diagrams. *Example 1*



1. Name a pair of adjacent complementary angles.

2. Name a pair of adjacent supplementary angles.

3. Name a pair of nonadjacent supplementary angles.

4. Name a pair of nonadjacent complementary angles.

In Exercises 5–8, find the angle measure. *Example 2*

5. $\angle 1$ is a complement of $\angle 2$, and $m\angle 1 = 23°$. Find $m\angle 2$.

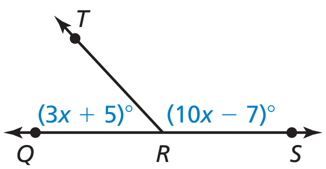
6. $\angle 3$ is a complement of $\angle 4$, and $m\angle 3 = 46°$. Find $m\angle 4$.

7. $\angle 5$ is a supplement of $\angle 6$, and $m\angle 5 = 78°$. Find $m\angle 6$.

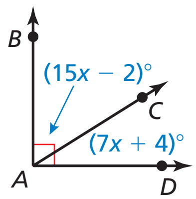
8. $\angle 7$ is a supplement of $\angle 8$, and $m\angle 7 = 109°$. Find $m\angle 8$.

In Exercises 9–12, find the measure of each angle.
Example 3

9.



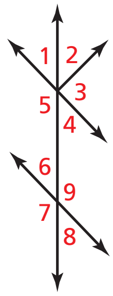
10.



11. $\angle UVW$ and $\angle XYZ$ are complementary angles, $m\angle UVW = (x - 10)°$, and $m\angle XYZ = (4x - 10)°$.

12. $\angle EFG$ and $\angle LMN$ are supplementary angles, $m\angle EFG = (3x + 17)°$, and $m\angle LMN = \left(\frac{1}{2}x - 5\right)°$.

In Exercises 13–16, use the diagram. *Example 4*



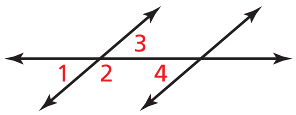
13. Identify all the linear pairs that include $\angle 1$.

14. Identify all the linear pairs that include $\angle 7$.

15. Are $\angle 6$ and $\angle 8$ vertical angles? Explain your reasoning.

16. Are $\angle 2$ and $\angle 5$ vertical angles? Explain your reasoning.

ERROR ANALYSIS In Exercises 17 and 18, describe and correct the error in identifying pairs of angles in the diagram.



17. ✗ $\angle 2$ and $\angle 4$ are adjacent angles.

18. ✗ $\angle 1$ and $\angle 3$ are a linear pair.

In Exercises 19–24, find the measure of each angle.
Example 5

19. Two angles form a linear pair. The measure of one angle is twice the measure of the other angle.

20. Two angles form a linear pair. The measure of one angle is $\frac{1}{3}$ the measure of the other angle.

21. The measure of an angle is $\frac{1}{4}$ the measure of its complement.

22. The measure of an angle is nine times the measure of its complement.

23. The ratio of the measure of an angle to the measure of its complement is 4 : 5.

24. The ratio of the measure of an angle to the measure of its complement is 2 : 7.

GO DIGITAL

MODELING REAL LIFE **In Exercises 25 and 26, the picture shows the Alamillo Bridge in Seville, Spain. In the picture, $m\angle 1 = 58^\circ$ and $m\angle 2 = 24^\circ$.**

25. Find the measure of the supplement of $\angle 1$.

26. Find the measure of the supplement of $\angle 2$.

27. **MODELING REAL LIFE** The foul lines of a baseball field intersect at home plate to form a right angle. A batter hits a fair ball such that the path of the baseball forms an angle of 27° with the third base foul line. What is the measure of the angle between the first base foul line and the path of the baseball?

28. **COLLEGE PREP** The arm of a crossing gate moves 42° from a vertical position. How many more degrees does the arm have to move so that it is horizontal?

Ⓐ 42° Ⓑ 138°

Ⓒ 48° Ⓓ 90°

29. **CONSTRUCTION** Construct a linear pair where one angle measure is 115°.

30. **CONSTRUCTION** Construct a pair of adjacent angles that have angle measures of 45° and 97°.

CONNECTING CONCEPTS **In Exercises 31–34, write and solve an algebraic equation to find the measure of each angle described.**

31. The measure of an angle is 6° less than the measure of its complement.

32. The measure of an angle is 12° more than twice the measure of its complement.

33. The measure of an angle is 3° more than $\frac{1}{2}$ the measure of its supplement.

34. Two angles form a linear pair. The measure of one angle is 15° less than $\frac{2}{3}$ the measure of the other angle.

35. **COLLEGE PREP** $m\angle U = 2x^\circ$, and $m\angle V = 4(m\angle U)$. Which value of x makes $\angle U$ and $\angle V$ complements of each other?

Ⓐ 25 Ⓑ 9 Ⓒ 36 Ⓓ 18

36. **HOW DO YOU SEE IT?** Determine whether you can conclude each statement from the diagram. Explain your reasoning.

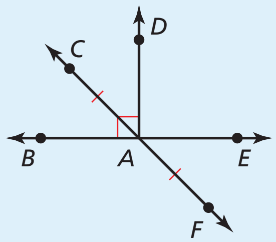
a. $\overline{CA} \cong \overline{AF}$

b. Points C, A, and F are collinear.

c. $\angle CAD \cong \angle EAF$

d. $\overline{BA} \cong \overline{AE}$

e. $\angle DAE$ is a right angle.



CRITICAL THINKING **In Exercises 37–42, tell whether the statement is *always*, *sometimes*, or *never* true. Explain your reasoning.**

37. Complementary angles are adjacent.

38. Angles in a linear pair are supplements of each other.

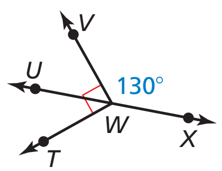
39. Vertical angles are adjacent.

40. Vertical angles are supplements of each other.

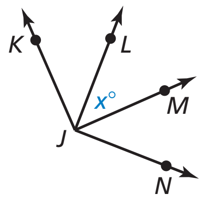
41. If an angle is acute, then its complement is greater than its supplement.

42. If two complementary angles are congruent, then the measure of each angle is 45°.

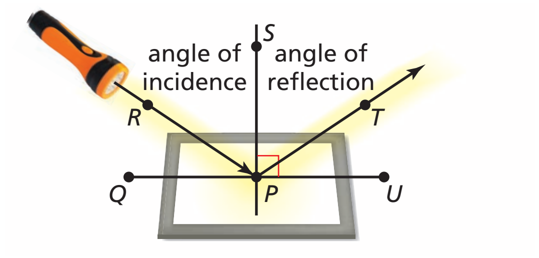
43. **CONNECTING CONCEPTS** Use the diagram. You write the measures of $\angle TWU$, $\angle TWX$, $\angle UWV$, and $\angle VWX$ on separate pieces of paper and place the pieces of paper in a box. You choose two pieces of paper out of the box at random. Find the probability that the angle measures you choose represent supplementary angles. Explain.



44. MP **REASONING** $\angle KJL$ and $\angle LJM$ are complements, and $\angle MJN$ and $\angle LJM$ are complements. Can you show that $\angle KJL \cong \angle MJN$? Explain your reasoning.

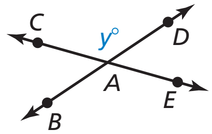


45. **MAKING AN ARGUMENT** Light from a flashlight strikes a mirror and is reflected so that the angle of reflection is congruent to the angle of incidence. Your classmate claims that $\angle QPR$ is congruent to $\angle TPU$ regardless of the measure of $\angle RPS$. Is your classmate correct? Explain your reasoning.



46. **THOUGHT PROVOKING**
Sketch a real-life situation that shows supplementary, complementary, and vertical angles.

47. **DRAWING CONCLUSIONS**
Use the diagram.



a. Write expressions for the measures of $\angle BAE$, $\angle DAE$, and $\angle CAB$.

b. What do you notice about the measures of vertical angles? Explain your reasoning.

48. **CONNECTING CONCEPTS** Let $m\angle 1 = x°$, $m\angle 2 = y_1°$, and $m\angle 3 = y_2°$. $\angle 2$ is the complement of $\angle 1$, and $\angle 3$ is the supplement of $\angle 1$.

a. Write equations for y_1 as a function of x and y_2 as a function of x. What is the domain of each function? Explain.

b. Graph each function and find its range.

49. **CONNECTING CONCEPTS** The sum of the measures of two complementary angles is 74° greater than the difference of their measures. Find the measure of each angle. Explain how you found the angle measures.

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REVIEW & REFRESH

WATCH

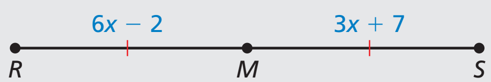
In Exercises 50 and 51, find the area of the polygon with the given vertices.

50. $K(-3, 4)$, $L(1, 4)$, $M(-4, -2)$, $N(0, -2)$

51. $X(-1, 2)$, $Y(-1, -3)$, $Z(4, -3)$

52. The midpoint of $\overline{JK}$ is $M(0, 1)$. One endpoint is $J(-6, 3)$. Find the coordinates of endpoint K.

53. Identify the segment bisector of $\overline{RS}$. Then find RS.



In Exercises 54–57, solve the equation. Graph the solution(s), if possible.

54. $|t + 5| = 3$

55. $\left|\frac{1}{4}d - 1\right| + 2 = 5$

56. $-4|7 + 2n| = 12$

57. $|-1.6q| = 7.2$

In Exercises 58 and 59, find the product.

58. $(8x^2 - 16 + 3x^3)(-4x^5)$

59. $(4s - 3)(7s + 5)$

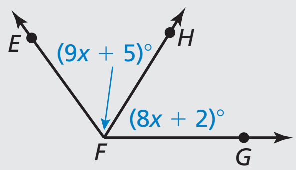
60. **MODELING REAL LIFE** The total cost (in dollars) of renting a cabin for n days is represented by the function $f(n) = 75n + 200$. The daily rate is doubled. The new total cost is represented by the function $g(n) = f(2n)$. Describe the transformation from the graph of f to the graph of g.

In Exercises 61 and 62, find the slope and the *y*-intercept of the graph of the linear equation.

61. $y = -5x + 2$

62. $y + 7 = \frac{3}{2}x$

63. Given that $m\angle EFG = 126°$, find $m\angle EFH$ and $m\angle HFG$.



In Exercises 64 and 65, find the angle measure.

64. $\angle 1$ is a supplement of $\angle 2$, and $m\angle 1 = 57°$. Find $m\angle 2$.

65. $\angle 3$ is a complement of $\angle 4$, and $m\angle 4 = 34°$. Find $m\angle 3$.

1 Chapter Review WITH CalcChat®

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Chapter Learning Target Understand basics of geometry.

Chapter Success Criteria
- ◆ I can name points, lines, and planes.
- ◆ I can measure segments and angles.
- ■ I can use formulas in the coordinate plane.
- ■ I can construct segments and angles.

◆ Surface
■ Deep

SELF-ASSESSMENT

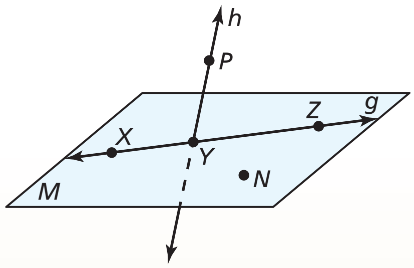
1 I do not understand. 2 I can do it with help. 3 I can do it on my own. 4 I can teach someone else.

1.1 Points, Lines, and Planes (pp. 3–10)

WATCH

Learning Target: Use defined terms and undefined terms.

Use the diagram.



1. Give another name for plane M.
2. Name a line in plane M.
3. Name a line intersecting plane M.
4. Name two rays.
5. Name a pair of opposite rays.
6. Name a point not in plane M.
7. Is it possible for the intersection of two planes to be a segment? a line? a ray? Sketch the possible situations.

Vocabulary VOCAB
undefined terms
point
line
plane
collinear points
coplanar points
defined terms
line segment, or segment
endpoints
ray
opposite rays
intersection

1.2 Measuring and Constructing Segments (pp. 11–18)

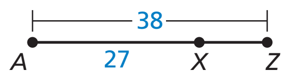
WATCH

Learning Target: Measure and construct line segments.

Find XZ**.**

8.

9.



10. Plot $A(8, -4)$, $B(3, -4)$, $C(7, 1)$, and $D(7, -3)$ in a coordinate plane. Then determine whether $\overline{AB}$ and $\overline{CD}$ are congruent.

11. You pass by school and the library on a walk from home to the bookstore, as shown below. How far from school is the library? How long does it take you to walk from home to the bookstore at an average speed of 68 meters per minute?

Vocabulary VOCAB
postulate
axiom
coordinate
distance between two points
construction
congruent segments
between

GO DIGITAL

1.3 Using Midpoint and Distance Formulas *(pp. 19–26)*

WATCH

Learning Target: Find midpoints and lengths of segments.

Vocabulary
midpoint
segment bisector

The endpoints of $\overline{ST}$ are given. Find the coordinates of the midpoint M. Then find the length of $\overline{ST}$.

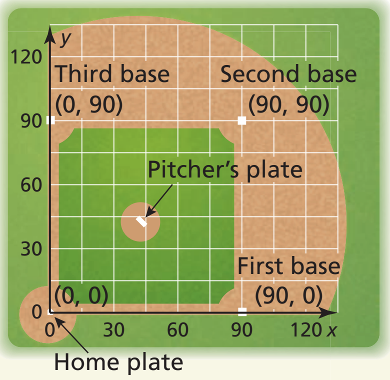
12. $S(-2, 4)$ and $T(3, 9)$

13. $S(6, -3)$ and $T(7, -2)$

14. The midpoint of $\overline{JK}$ is $M(6, 3)$. One endpoint is $J(14, 9)$. Find the coordinates of endpoint K.

15. Point M is the midpoint of $\overline{AB}$, where $AM = 3x + 8$ and $MB = 6x - 4$. Find AB.

16. The coordinate plane shows distances (in feet) on a baseball infield. The pitcher's plate is about 3 feet closer to home plate than the midpoint between home plate and second base is to home plate. Estimate the distance between home plate and the pitcher's plate. Explain how you found your answer.



17. The endpoints of $\overline{DE}$ are $D(-3, y)$ and $E(x, 6)$. The midpoint of $\overline{DE}$ is $M(4, 2)$. What is the length of $\overline{DE}$?

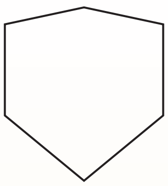
1.4 Perimeter and Area in the Coordinate Plane *(pp. 27–34)*

WATCH

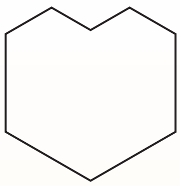
Learning Target: Find perimeters and areas of polygons in the coordinate plane.

Classify the polygon by the number of sides. Tell whether it is *convex* or *concave*.

18.



19.

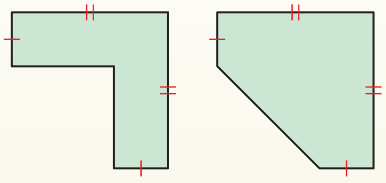


Find the perimeter and area of the polygon with the given vertices.

20. $W(5, 6)$, $X(5, -1)$, $Y(2, -1)$, $Z(2, 6)$

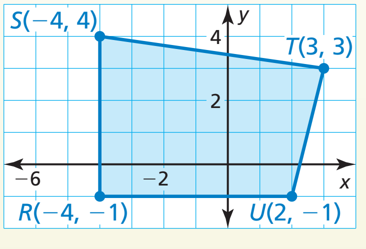
21. $E(6, -2)$, $F(6, 5)$, $G(-1, 5)$

22. Two polygons are shown. Compare the area and perimeter of the concave polygon with the area and perimeter of the convex polygon.



23. Find the perimeter of quadrilateral $RSTU$ in the coordinate plane at the right.

24. Find the area of quadrilateral $RSTU$.



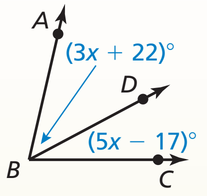
1.5 Measuring and Constructing Angles (pp. 35–44)

WATCH

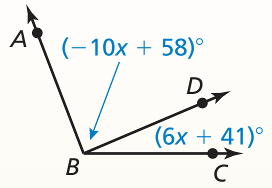
Learning Target: Measure, construct, and describe angles.

Find $m\angle ABD$ and $m\angle CBD$.

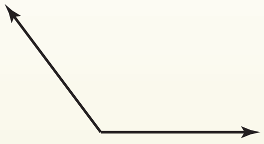
25. $m\angle ABC = 77°$



26. $m\angle ABC = 111°$



27. Find the measure of the angle using a protractor.



28. Given that P is in the interior of $\angle ABC$, $m\angle PBC = 30°$, $\angle DBC$ is a right angle, and $m\angle ABD = 26°$, what are the possible measures of $\angle ABP$?

Vocabulary

VOCAB

angle
vertex
sides of an angle
interior of an angle
exterior of an angle
measure of an angle
acute angle
right angle
obtuse angle
straight angle
congruent angles
angle bisector

GO DIGITAL

1.6 Describing Pairs of Angles (pp. 45–52)

WATCH

Learning Target: Identify and use pairs of angles.

$\angle 1$ and $\angle 2$ are complementary angles. Given $m\angle 1$, find $m\angle 2$.

29. $m\angle 1 = 12°$

30. $m\angle 1 = 83°$

$\angle 3$ and $\angle 4$ are supplementary angles. Given $m\angle 3$, find $m\angle 4$.

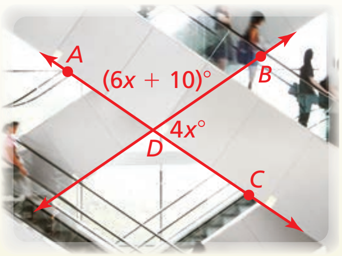
31. $m\angle 3 = 116°$

32. $m\angle 3 = 56°$

33. Construct a linear pair, where one angle measure is 35°. Label the measures of both angles.

34. The measure of an angle is 4 times the measure of its supplement. Find the measures of both angles.

35. Find the measures of $\angle ADB$ and $\angle BDC$ formed between the escalators shown at the right.



Vocabulary

VOCAB

adjacent angles
complementary angles
supplementary angles
linear pair
vertical angles

Mathematical Practices

Make Sense of Problems and Persevere in Solving Them

Mathematically proficient students plan a solution pathway rather than simply jumping into a solution attempt.

1. In Exercise 36 on page 25, why is it necessary to understand the problem and make a plan before solving? How does stating the given information and describing how it is related help you make a plan to solve the problem?

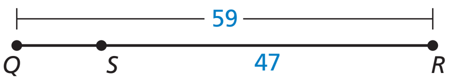
2. Describe the plan you used to find the area of the lion sanctuary in Exercise 24 on page 33.

1 Practice Test WITH CalcChat®

Find *QS*. Explain how you found your answer.

1.

2.



3. The endpoints of $\overline{AB}$ are $A(-4, -8)$ and $B(-1, 4)$. Find the coordinates of the midpoint M. Then find the length of $\overline{AB}$.

4. The midpoint of $\overline{EF}$ is $M(1, -1)$. One endpoint is $E(-3, 2)$. Find the coordinates of endpoint F.

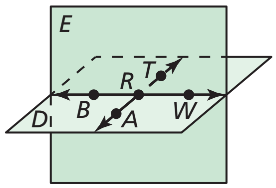
Use the diagram to decide whether the statement is true or false.

5. Points A, R, and B are collinear.

6. $\overleftrightarrow{BW}$ and $\overleftrightarrow{AT}$ are lines.

7. $\overrightarrow{RB}$ and $\overrightarrow{RT}$ are opposite rays.

8. Plane D could also be named plane ART.



Find the perimeter and area of the polygon with the given vertices.

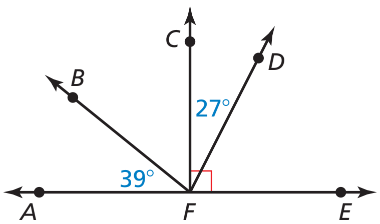
9. $P(-3, 4)$, $Q(1, 4)$, $R(3, -2)$, $S(-1, -2)$

10. $J(-1, 3)$, $K(5, 3)$, $L(2, -2)$

11. $\overrightarrow{BX}$ bisects $\angle ABC$ to form two congruent acute angles. What type of angle can $\angle ABC$ be?

12. Given $\angle RST$, U is in the interior of the angle, $m\angle TSU$ is 6° less than 5 times $m\angle RSU$, and $m\angle RST = 48°$. Write and solve a system of equations to find $m\angle RSU$ and $m\angle TSU$.

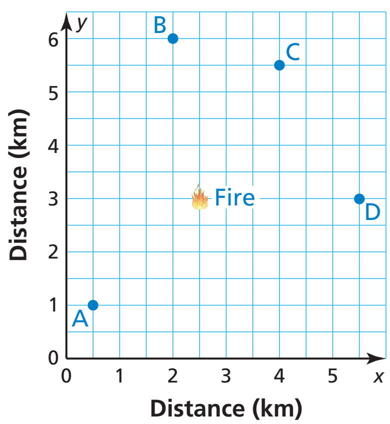
13. In the diagram at the right, identify all supplementary and complementary angles. Explain. Then find $m\angle DFE$, $m\angle BFC$, and $m\angle BFE$.



14. Sketch a figure that contains a plane and two lines that all intersect at one point.

15. Your community decides to install a rectangular swimming pool in a park. There is a 48-foot by 81-foot rectangular area available. There must be at least a 3-foot border around the pool. Draw a diagram of this situation in a coordinate plane. Based on the constraints, find the perimeter and the area of the largest swimming pool possible.

16. Four wildland firefighting crews are approaching a small, growing fire, as shown. The crews are at the positions labeled A, B, C, and D. Which position is closest to the fire?



1 Performance Task
Eye of the Tiger

Conflict between humans and wild animals is a major threat to both humans and animals.

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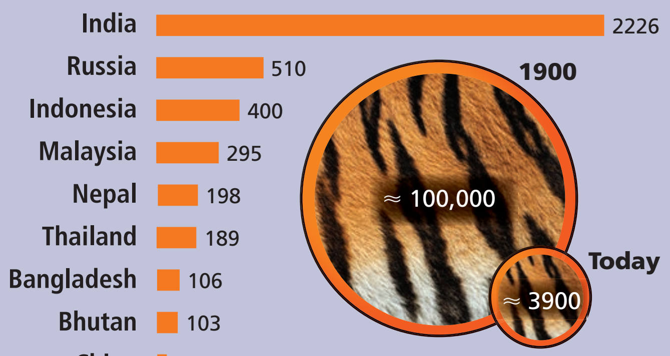
Causes of human-tiger conflict include the following:

- **HABITAT AVAILABILITY**
  Deforestation, habitat degradation, and increasing human populations force tigers and humans into closer proximity.
- **WILD PREY AVAILABILITY**
  Prey species are diminished by overexploitation and competition with livestock. A low density of wild prey increases the chance of human-tiger conflict.
- **SOCIOECONOMIC FACTORS**
  Attitudes, perceptions, beliefs, education, and economic situations affect views on how to interact with tigers.
- **IMPROPER LIVESTOCK MANAGEMENT**
  Herding practices and locations of grazing pastures can leave livestock susceptible to attacks.
- **HUMAN BEHAVIOR**
  Baiting or hunting tigers, and sleeping in exposed locations increase the risk of an attack.

Estimated Wild Tiger Populations



The wild tiger population has decreased about 96% since 1900.

WILDLIFE RESERVATION

INFO

You propose a new wildlife reservation in an attempt to limit human-tiger conflict. Use points and line segments to sketch the outline of your reservation in a coordinate plane. Name each point and line segment in your sketch.

A local government requires several details before considering your proposal. Provide the following information:

- the length of each side of the reservation
- the area of the reservation
- the measures of the angles formed by the sides of the reservation
- the coordinates of at least three gates, located at midpoints of the sides of the reservation

1 College and Career Readiness WITH CalcChat®

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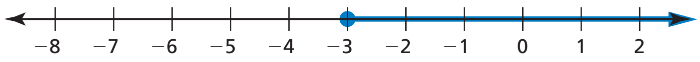
WATCH Tutorial videos are available for each exercise.

1. Which inequality is represented by the graph?



Ⓐ $x \leq -3$

Ⓑ $x < -3$

Ⓒ $x \geq -3$

Ⓓ $x > -3$

2. Order the terms so that each consecutive term builds off the previous term.

plane	segment	line	point	ray

3. The endpoints of a line segment are $(-6, 13)$ and $(11, 5)$. Which of the following is the midpoint and the length of the segment?

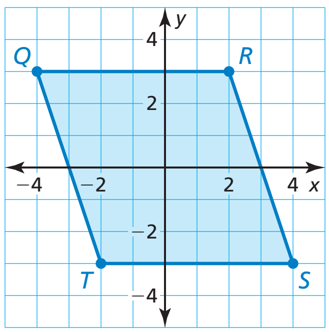
Ⓐ $\left(\frac{5}{2}, 4\right)$; $\sqrt{353}$ units

Ⓑ $\left(\frac{5}{2}, 9\right)$; $\sqrt{353}$ units

Ⓒ $\left(\frac{5}{2}, 4\right)$; $\sqrt{89}$ units

Ⓓ $\left(\frac{5}{2}, 9\right)$; $\sqrt{89}$ units

4. Which of the following is the perimeter and area of the figure shown?



Ⓐ $6 + 2\sqrt{10}$ units; 36 square units

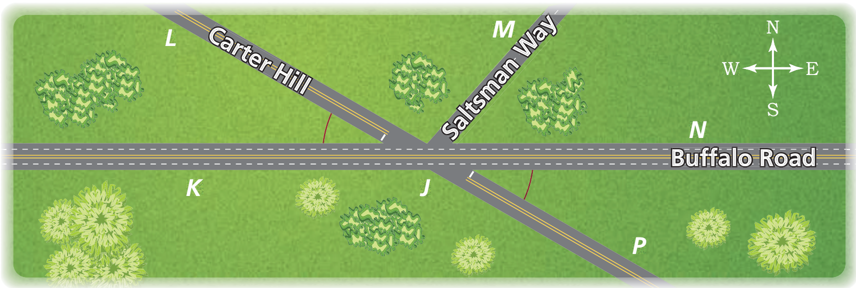
Ⓑ $6 + 2\sqrt{10}$ units; $12\sqrt{10}$ square units

Ⓒ $12 + 4\sqrt{10}$ units; 36 square units

Ⓓ $12 + 4\sqrt{10}$ units; $12\sqrt{10}$ square units

5. Plot the points $W(-1, 1)$, $X(5, 1)$, $Y(5, -2)$, and $Z(-1, -2)$ in a coordinate plane. What type of polygon do the points form? Your friend claims that you could use this polygon to represent a basketball court with an area of 4050 square feet and a perimeter of 270 feet. Do you support your friend's claim? Explain.

6. Three roads come to an intersection point that the people in your town call Five Corners, as shown in the figure.



Answer parts (a) through (c) using the angles given below.

a. You are traveling west on Buffalo Road and turn left onto Carter Hill. What is the name of the angle through which you turn?

b. Identify all the vertical angles.

c. Identify all the linear pairs.

$\angle KJL$	$\angle KJM$	$\angle KJN$	$\angle KJP$	$\angle LJM$
$\angle LJN$	$\angle LJP$	$\angle MJN$	$\angle MJP$	$\angle NJP$

7. What is the nth term of the geometric sequence $-3, 6, -12, 24, -48, \ldots$?

Ⓐ $a_n = -3(-2)^{n-1}$

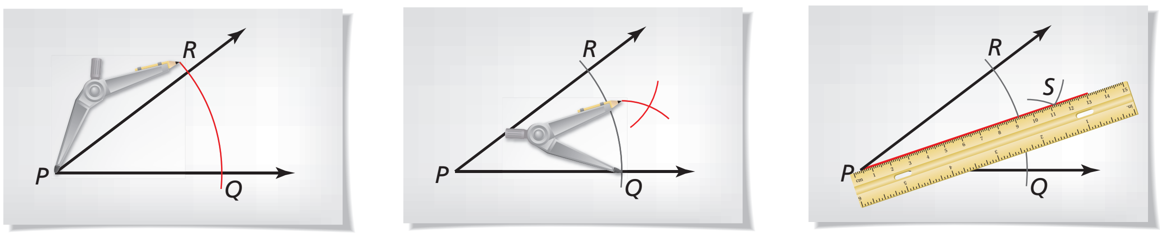
Ⓑ $a_n = -2(-3)^{n-1}$

Ⓒ $a_n = -3(2)^{n-1}$

Ⓓ $a_n = -3\left(-\frac{1}{2}\right)^{n-1}$

8. Use the steps in the construction to explain how you know that $\overrightarrow{PS}$ is the angle bisector of $\angle RPQ$.

Step 1 **Step 2** **Step 3**



9. Which factorization can be used to find the zeros of the function $f(x) = 15x^2 + 14x - 8$?

Ⓐ $f(x) = x(15x + 14) - 8$

Ⓑ $f(x) = 15x^2 + 2(7x - 4)$

Ⓒ $f(x) = (5x + 8)(3x - 1)$

Ⓓ $f(x) = (5x - 2)(3x + 4)$